Moving Mesh Method with Conservative Interpolation Based on L^2-Projection

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Abstract. We develop an efficient one-dimensional moving mesh algorithm for solving partial differential equations. The main contribution of this paper is to design an effective interpolation scheme based on L^2-projection for the moving mesh method. The proposed method preserves not only the mass-conservation but also the first order momentum of the underlying numerical solution at each mesh redistribution step. Numerical examples are presented to demonstrate the effectiveness of the new interpolation technique.

Key words: Moving mesh method; conservative interpolation; L^2-projection.

1   Introduction

In the past two decades, there has been important progress in developing adaptive mesh methods for PDEs. Mesh adaptivity is usually of two types: local mesh refinement and moving mesh method. In local mesh refinement methods, see, e.g., [1], an adaptive mesh is obtained by adding or removing grid points to achieve a desired level of accuracy. They allow a posteriori error estimates to guide the mesh refinement and coarsening. However, local mesh refinement methods require complicated data structures, and increase computer code complexity. In the moving mesh method, the mesh points are moved continuously in the whole domain to concentrate in regions where the solution has the largest variations. Unlike the local mesh refinement methods, the moving mesh method keeps a fixed number of mesh points throughout the computation.

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There have been a variety of moving mesh techniques, including the variational approach [2, 21]; the moving finite element methods [7, 13]; the moving mesh PDEs approach [4, 5, 10]; the moving mesh methods based on harmonic mapping of [9, 11, 12]. We can classify the moving mesh methods into two classes: interpolation free moving mesh methods and moving mesh methods using interpolation. The earlier works such as [8, 13] are directly based on the equidistribution principle which was first proposed by de Boor [3]. Generally speaking, the earlier moving mesh methods have a common feature that the grid equation is strongly coupled with the underlying PDEs, and consequently, their numerical approximation leads to a nonlinear system even if the underlying PDEs are linear. Since the mesh redistribution and the PDEs are solved simultaneously, the interpolation of solution information from the old mesh to the new mesh is unnecessary. So we call this class of methods interpolation-free moving mesh methods. Another group of interpolation-free moving mesh methods was developed by Russell and his research group; see, e.g., [4]. Their moving mesh equation is written as moving mesh partial differential equations (MMPDEs) based on the equidistribution principle.

Li et al. [11] developed a moving finite element method containing two independent parts: solution evolution and mesh redistribution. These two parts are relatively independent in the sense that the change of the underlying PDEs will only affect the first part (solution evolution), while the second part (mesh redistribution) is kept unchanged. An immediate advantage of this class of methods is the computer code simplicity. In the mesh redistribution part, the adaptive mesh is usually generated through some iterative algorithm, which makes the solution interpolation from one iteration step to the next indispensable. In the moving finite element paper [11], a convection equation is solved to pass the solution in the old mesh to the new mesh. Later, Tang and Tang [18] proposed a moving mesh finite volume method, which uses conservative interpolation to pass the soution information from the old mesh to the new mesh. This method has recently been applied to the Hamilton-Jacobi equations [19], convection-dominated problems [24, 25], and phase-field problems [17]. The conventional interpolation method, such as third order polynomial interpolation [14], is applied to solve the Schrödinger equations involving static blow-ups.

The main objective of this work is to propose an interpolation algorithm based on L^2-projection for the moving mesh method. Similar to [11, 18, 25], our moving mesh approach includes two independent parts: PDE solution evolution and mesh redistribution. The first part is problem-dependent, that is to say, we need to design appropriate numerical solvers for different governing equations. In the second part, we need to transfer the solution information from the old mesh to the new one. This is similar to an Arbitrary Lagrangian-Eulerian (ALE) method, where a remapping algorithm plays an essential and important role. A good remapping scheme should possess properties such as accuracy, conservation and computational efficiency. Many authors have worked on developing remapping strategies. Recently, Cheng and Shu [6] proposed a high-order accurate conservative remapping method on staggered meshes. Their method is built on the framework of ENO interpolation and reconstruction, so it has the good properties such as high-order