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A Unified Gas Kinetic Scheme for Continuum and Rarefied Flows V: Multiscale and Multi-Component Plasma Transport

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Abstract. As a continuation of developing multiscale method for the transport phenomena, a unified gas kinetic scheme (UGKS) for multi-scale and multi-component plasma simulation is constructed. The current scheme is a direct modeling method, where the time evolution solutions from the Vlasov-BGK equations of electron and ion and the Maxwell equations are used to construct a scale-dependent plasma simulation model. The modeling scale used in the UGKS is the mesh size scale, which can be comparable to or much larger than the local mean free path. As a result, with the variation of modeling scales in space and time through the so-called cell's Knudsen number and normalized Larmor radius, the discretized governing equations can recover a wide range of plasma evolution from the Vlasov equation in the kinetic scale to different-type of magnetohydrodynamic (MHD) equations in the hydrodynamic scale. The UGKS provides a general evolution model, which goes to the Vlasov equation in the kinetic scale and many types of MHD equations in the hydrodynamic scale, such as the two fluids model, the Hall, the resistive, and the ideal MHD equations. All current existing governing equations become the subsets of the UGKS, and the UGKS bridges these distinguishable governing equations seamlessly. The construction of UGKS is based on the implementation of physical conservation laws and the un-splitting treatment of particle collision, acceleration, and transport in the construction of a scale-dependent numerical flux across a cell interface. At the same time, the discretized plasma evolution equations are coupled with the Maxwell equations for electro-magnetic fields, which also cover a scale-dependent transition between the Ampère's law and the Ohm's law for the calculation of electric field. The time step of UGKS is not limited by the relaxation time, the cyclotron period, and the speed of light in the ideal-MHD regime. Our scheme is able to give a physically accurate solution for plasma simulation with a wide range of Knudsen number and normalized Larmor radius. It can be used to study the phenomena from the Vlasov limit to the

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scale of plasma skin depth for the capturing of two-fluid effect, and the phenomena in the plasma transition regime with a modest Knudsen number and Larmor radius. The UGKS is validated by numerical test cases, such as the Landau damping and two stream instability in the kinetic regime, and the Brio-Wu shock tube problem, and the Orszag-Tang MHD turbulence problem in the hydrodynamic regime. The scheme is also used to study the geospace environment modeling (GEM), such as the challenging magnetic reconnection problem in the transition regime. At the same time, the magnetic reconnection mechanism of the Sweet-Parker model and the Hall effect model can be connected smoothly through the variation of Larmor radius in the UGKS simulations. Overall, the UGKS is a physically reliable multi-scale plasma simulation method, and it provides a powerful and unified approach for the study of plasma physics.

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Key words: Unified gas-kinetic scheme, plasma, Vlasov equation, two-fluid equations, MHD equations, magnetic field reconnection.

Nomenclature

Constants

ϵ_0	Vacuum permittivity
μ_0	Vacuum permeability
c	Speed of light in vacuum
e	Charge of a proton

Characteristic variables

B_0	Characteristic strength of magnetic field
l_0	Characteristic length, plasma size
m_i	Characteristic molecular mass, ion molecular mass
n_0	Characteristic number density
U_0	Characteristic velocity, ion thermal velocity

Sup/Sub-scripts

α	Components of plasma: $\alpha=i$ stands for ion, $\alpha=e$ stands for electron
$\hat{\cdot}$	Nondimensional quantities
$\bar{\cdot}$	Averaged quantities in AAP model
i,j	Index for numerical cell and numerical cell interface