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A Second-Order Cell-Centered Lagrangian Method for Two-Dimensional Elastic-Plastic Flows

Jun-Bo Cheng[1], Yueling Jia[1,*], Song Jiang[1], Eleuterio F. Toro[2] and Ming Yu[1]

[1] *Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing, 100094, China.*
[2] *Laboratory of Applied Mathematics, University of Trento, Trento, Italy.*

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Abstract. For 2D elastic-plastic flows with the hypo-elastic constitutive model and von Mises' yielding condition, the non-conservative character of the hypo-elastic constitutive model and the von Mises' yielding condition make the construction of the solution to the Riemann problem a challenging task. In this paper, we first analyze the wave structure of the Riemann problem and develop accordingly a **F**our-**R**arefaction wave approximate **R**iemann **S**olver with **E**lastic waves (**FRRSE**). In the construction of FRRSE one needs to use an iterative method. A direct iteration procedure for four variables is complex and computationally expensive. In order to simplify the solution procedure we develop an iteration based on two nested iterations upon two variables, and our iteration method is simple in implementation and efficient. Based on FRRSE as a building block, we propose a 2nd-order cell-centered Lagrangian numerical scheme. Numerical results with smooth solutions show that the scheme is of second-order accuracy. Moreover, a number of numerical experiments with shock and rarefaction waves demonstrate the scheme is essentially non-oscillatory and appears to be convergent. For shock waves the present scheme has comparable accuracy to that of the scheme developed by Maire et al., while it is more accurate in resolving rarefaction waves.

AMS subject classifications: 35L50, 35Q74, 74C05, 74F10, 74M20

Key words: Cell-centered Lagrangian scheme, high-order scheme, hypo-elastic constitutive model, four-rarefaction Riemann solver with elastic waves.

1 Introduction

This paper aims at the construction of a high-order cell-centered Lagrangian scheme for elastic-plastic flows in two-spatial dimensions with the isotropic elastic-plastic model,

*Corresponding author. *Email addresses:* `jia_yueling@iapcm.ac.cn` (Y. L. Jia), `cheng_junbo@iapcm.ac.cn` (J.-B. Cheng), `jiang@iapcm.ac.cn` (S. Jiang), `eleuterio.toro@unitn.it` (E. F. Toro), `Yu_ming@iapcm.ac.cn` (M. Yu)

initially developed by Wilkins [1]. The scheme is constructed based on an approximate Riemann solver for the equations of the elastic-plastic solids. In Wilkins' model, a perfectly elastic material is characterized by Hooke's law in terms of an incremental strain resulting in an incremental stress and Von Mises' yielding condition is used to describe the elastic limit. The earliest simulation for elastic-plastic hydrodynamic equations with Wilkins' model was developed by Wilkins in 1962 [1], in which the equations of momentum and specific internal energy are discretized on a staggered grid and artificial viscosity is employed to simulate moving shocks in order to damp spurious numerical oscillations. Besides the staggered Lagrangian approach, Eulerian and cell-centered Lagrangian schemes have also been used in the simulation of elastic-plastic flows.

Eulerian methods [8]- [12] are suitable for problems involving discontinuous waves and large deformations. However, most of them are for problems with the hyper-elastic model for isotropic materials. Compared with staggered Lagrangian schemes, cell-centered Lagrangian schemes have their own advantages. For the cell-centered Lagrangian scheme it is not necessary to use artificial viscosity and the scheme is conservative because the equation for total energy conservation, not the specific internal energy, is discretised. In recent years, the cell-centered Lagrangian scheme has attracted much attention [2]- [5], where cell-centered Lagrangian schemes are not only constructed for the hyper-elastic models, but also for hypo-elastic models. In these papers, a nodal Riemann solver is deduced from conservation of the total energy, but the structure of the Riemann solution is not fully exploited. Some authors, such as in [6] and [7], have utilized the structure of the solution of the Riemann problem to construct Riemann solvers for the governing equations with hypo-elastic models. Gavrilyuk et al. [7] analyzed the structure of the solution to the Riemann problem and developed a Riemann solver for the hyperbolic nonconservative model with a system of linear elasticity for transverse waves. Besides, the elastic energy is included in the total energy and an extra evolution equation is added in order to make the elastic transform reversible in the absence of shock waves. Després [6] constructed a shock solution to the non-conservative equations with the hypo-elasticity model and found that a sonic point was necessary to give a compression solution that begins at a constrained compressed state.

In this paper, we first analyze the structure of the solution of the Riemann problem for the governing equations with hypo-elastic models in the normal direction to a generic cell edge in two-spatial dimensions, then construct a four-rarefaction approximate Riemann solver with elastic waves (FRRSE) for the non-conservative equations of Wilkins' model with von Mises' yielding condition. Moreover, we use our FRRSE to define a numerical flux to construct a high-order cell-centered Lagrangian scheme.

The paper is organized as follows. We describe the governing equations in two-spatial dimensions in Section 2, our FRRSE is introduced in Section 3 and an associated high order cell-centered Lagrangian scheme for non-conservative elastic-plastic flows is developed in Section 4. A number of numerical examples are presented in Section 5 to validate our scheme, and conclusions follow in Section 6.