

# Derivation of a Multilayer Approach to Model Suspended Sediment Transport: Application to Hyperpycnal and Hypopycnal Plumes

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**Abstract.** We propose a multi-layer approach to simulate hyperpycnal and hypopycnal plumes in flows with free surface. The model allows to compute the vertical profile of the horizontal and the vertical components of the velocity of the fluid flow. The model can describe as well the vertical profile of the sediment concentration and the velocity components of each one of the sediment species that form the turbidity current. To do so, it takes into account the settling velocity of the particles and their interaction with the fluid. This allows to better describe the phenomena than a single layer approach. It is in better agreement with the physics of the problem and gives promising results. The numerical simulation is carried out by rewriting the multi-layer approach in a compact formulation, which corresponds to a system with non-conservative products, and using path-conservative numerical scheme. Numerical results are presented in order to show the potential of the model.

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## 1 Introduction

When a river that carries sediment in suspension enters into lake or the ocean, it can form a plume that advects the sediment from the river mouth. Based on the difference of density [27], these particle-bearing flows are said to be ‘hypopycnal’ (or an ‘overflow’) if the

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combined density of the sediment and interstitial fluid is lower than that of the ambient. If the combined density is higher than that of the ambient, it is said to be 'hyperpycnal' (or an 'underflow'). Hyperpycnal plumes are a class of sediment-laden gravity current commonly referred to as turbidity currents [20,24,27].

Turbidity currents play an important role in the in erosion and deposition over continental slopes and submarine canyons and have important effects on marine constructions and infrastructures near river mouths and continental shelves. Understanding the evolution of turbidity currents is of great importance.

Only limited observational records exist for the occurrence and flow of turbidity currents. This is due to the difficulty in predicting the time and frequency of turbidity currents as well as the destructive nature of such sediment-laden flows. As a result, most of our knowledge about these flows is derived from small scale laboratory experiments like the ones described in [1, 13, 16–18]. Given the lack of observations, numerical modeling is an excellent tool to gain an increased understanding of the evolution of turbidity currents.

Some layer-averaged models have been previously developed on the basis of small-scale tank experiments of particle-driven density currents in [5,8,15,17,23]. Although this layer-averaged approach gives a fast and valuable information, it has the disadvantage that the vertical distribution of the sediment in suspension is lost.

Due to the nature of the phenomena studied, losing this kind of information in the vertical direction is a mayor drawback. Nevertheless, a direct numerical simulation (DNS) of the fully 3d system would be computationally expensive. Moreover, additional difficulties arise, for instance, the treatment of the surface boundary is complex and shallow areas are in general poorly resolved.

One alternative to DNS simulation is to use models based on the bottom-following sigma coordinates (see [7] and references therein). Unfortunately, sigma coordinate models have their own drawbacks, associated with the evaluation of horizontal gradients of depth-dependent quantities such as density and velocity [14,25].

Another recent technique, based on a multilayer approach [2,3,11], has shown to be specially useful in order to generalize shallow water type models in order to keep track of the vertical components of the averaged variables in the classical shallow water equations. This technique has been applied in [12], where the authors propose an application of the multilayer approach to study polydisperse sedimentation.

In [12] a multilayer polydisperse sedimentation model was proposed following the technique introduced in [2]. In this paper we propose a different approach to derive the model. The multilayer is obtained using a vertical discontinuous Galerkin approach for which the vertical velocity and each sediment species involved in the turbidity current are supposed to be piecewise linear and the horizontal velocity is supposed to be piecewise constant. The mass and momentum transfer terms among the layers are obtained from the jump conditions of the conserved principles. The key point is the computation of the jump of the vertical velocity at each interface in terms of the jump condition associated to the definition of the mass transfer. Compared to the previous technique this new