An Adaptive Finite Element PML Method for the Acoustic-Elastic Interaction in Three Dimensions

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Abstract. Consider the scattering of a time-harmonic acoustic incident wave by a bounded, penetrable, and isotropic elastic solid, which is immersed in a homogeneous compressible air or fluid. The paper concerns the numerical solution for such an acoustic-elastic interaction problem in three dimensions. An exact transparent boundary condition (TBC) is developed to reduce the problem equivalently into a boundary value problem in a bounded domain. The perfectly matched layer (PML) technique is adopted to truncate the unbounded physical domain into a bounded computational domain. The well-posedness and exponential convergence of the solution are established for the truncated PML problem by using a PML equivalent TBC. An a posteriori error estimate based adaptive finite element method is developed to solve the scattering problem. Numerical experiments are included to demonstrate the competitive behavior of the proposed method.

1 Introduction

Consider the incidence of a time-harmonic acoustic wave onto a bounded, penetrable, and isotropic elastic solid, which is immersed in a homogeneous and compressible air or fluid. Due to the interaction between the incident wave and the solid obstacle, an elastic wave is excited inside the solid region, while the acoustic incident wave is scattered in the air/fluid region. This scattering phenomenon leads to an air/fluid-solid interaction problem. The surface of the elastic solid divides the whole three-dimensional space into a bounded interior domain and an open exterior domain where the elastic

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wave and the acoustic wave occupies, respectively. The two waves are coupled together on the surface via the interface conditions: continuity of the normal component of velocity and the continuity of traction. The acoustic-elastic interaction problems have received ever-increasing attention due to their significant applications in geophysics and seismology [22,23]. These problems have been examined mathematically by using either variational method [18, 19] or boundary integral equation method [24, 28]. Many computational approaches have also been developed to numerically solve these problems such as boundary element method [17,31] and coupling of finite and boundary element methods [16].

Since the work by Bérenger [4], the perfectly matched layer (PML) technique has been extensively studied and widely used to simulate various wave propagation problems, which include acoustic waves [5, 12, 21, 27, 32], elastic waves [6, 11, 13, 20, 26], and electromagnetic waves [3, 15]. The PML is to surround the domain of interest by a layer of finite thickness fictitious material which absorbs all the waves coming from inside the computational domain. It has been proven to be an effective approach to truncated open domains in the wave computation. Combined with the PML technique, the adaptive finite element method (FEM) has recently been developed to solve the diffraction grating problems [2,8,25] and the obstacle scattering problems [7,9,10]. Despite the large number of work done so far, they were concerned with a single wave propagation problem, i.e., either an acoustic wave, or an elastic wave, or an electromagnetic wave. It is very rare to study rigorously the PML problem for the interaction of multiple waves.

This paper aims to investigate the adaptive finite element PML method for solving the acoustic-elastic interaction problem. An exact transparent boundary condition (TBC) is developed to reduce the problem equivalently into a boundary value problem in a bounded domain. The PML technique is adopted to truncate the unbounded physical domain into a bounded computational domain. The variational approach is taken to incorporate naturally the interface conditions which couple the two waves. The well-posedness and exponential convergence of the solution are established for the truncated PML problem by using a PML equivalent TBC. The proofs rely on the error estimate between the two transparent boundary operators. To efficiently resolve the solution with possible singularities, the a posteriori error estimate based adaptive FEM is developed to solve the truncated PML problem. The error estimate consists of the PML error and the finite element discretization error, and provides a theoretical basis for the mesh refinement. Numerical experiments are reported to show the competitive behavior of the proposed method.

The paper is organized as follows. In Section 2, we introduce the model equations for the acoustic-elastic interaction problem. In Section 3, we present the PML formulation and prove the well-posedness and convergence of the solution for the truncated PML problem. In Section 4, we discuss the numerical implementation and show some numerical experiments. The paper is concluded with some general remarks in Section 5.