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A New Earthquake Location Method Based on the Waveform Inversion

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Abstract. In this paper, a new earthquake location method based on the waveform inversion is proposed. As is known to all, the waveform misfit function under the L^2 measure is suffering from the cycle skipping problem. This leads to a very small convergence domain of the conventional waveform based earthquake location methods. In present study, by introducing and solving two simple sub-optimization problems, we greatly expand the convergence domain of the waveform based earthquake location method. According to a large number of numerical experiments, the new method expands the range of convergence by several tens of times. This allows us to locate the earthquake accurately even from some relatively bad initial values.

AMS subject classifications: 49N45, 65K10, 86-08, 86A15

Key words: Computational seismology, inverse theory, waveform inversion, earthquake location.

1 Introduction

The earthquake location is a fundamental problem in seismology [11, 31]. It consists of two parts: the determination of hypocenter $\boldsymbol{\xi}$ and origin time τ. These information are extremely important in quantitative seismology, e.g. the earthquake early warning system [28], the investigation of seismic heterogeneous structure [35, 37]. In particular, there are also significant interests in micro-earthquake which has many applications in exploration seismology [18, 26]. It also has a similar mathematical framework with the other source localization or source identification problems, e.g. [1, 4, 5, 23].

Due to the importance of the earthquake location problem, numerous studies have been done theoretically and experimentally [11–13, 26, 30]. However, many studies are

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based on the ray theory, which has low accuracy when the wave length is not small enough compared to the scale of wave propagating region [9,15,27,40]. This may lead to inaccurate or even incorrect earthquake location results. An alternative way is to solve the wave equation directly to get accurate waveform information for inversion. This method is becoming popular in recent years, as a result of the fast developing of computational power and techniques [14,16,19–21,29,34].

In the work by [21], see also [16], the spectral-element solvers are implemented to invert the basic information of earthquakes. The misfit functions defined based upon the envelope of the waveforms are minimized to provide the best estimation of source model parameters. Another approach proposed by [35] is based on the wave-field relation between the hypocenter $\boldsymbol{\xi}$ and its perturbation $\boldsymbol{\xi}+\delta\boldsymbol{\xi}$ [3]. Due to the foregoing observation, the travel-time differences between the synthetic signal and the real signal can be approximately expressed as the linear function of hypocenter perturbation $\delta\boldsymbol{\xi}$. The authors then derived the sensitivity kernel by using the forward and adjoint wavefields.

However, the above mentioned papers on the earthquake location are not directly used the waveform difference since the waveform misfit function under the L^2 measure is suffering from the cycle skipping problem [21]. Consider the bad mathematical properties of the delta function $f(t-\tau)\delta(\boldsymbol{x}-\boldsymbol{\xi})$, who is appeared as the source of wave equation, even small perturbation of hypocenter $\delta\boldsymbol{\xi}$ and origin time $\delta\tau$ would generate large deviation of waveform. Thus, it is not surprising that the range of convergence of the conventional waveform based method is very small. On the other hand, the waveform signal may contain more information, which could lead to more accurate location result. Thus, it is necessary to develop new techniques to expand the convergence domain of the waveform based location method.

In this paper, we present a new method to locate the earthquake accurately. For the sake of simplicity, we use the acoustic wave equation and only deal with the earthquake hypocenter and origin time. There is no essential difficulty to consider the elastic wave equation or involve more earthquake information, e.g. the moment magnitudes [21]. The starting point is to keep $\frac{\|\delta s(\boldsymbol{x},t)\|}{\|s(\boldsymbol{x},t)\|} \ll 1$ in a modified sense. This is a fundamental assumption of the first-order Born approximation in the adjoint method. But it is not easy to guarantee in the classical sense, even if $\frac{\|\delta\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|}$ and $\frac{\|\delta\tau\|}{\|\tau\|}$ are small. This is due to the bad mathematical properties of the delta function $f(t-\tau)\delta(\boldsymbol{x}-\boldsymbol{\xi})$ in the wave equation. To solve this problem, we shift the synthetic data so that its difference with the real data is minimized. The shifting parameter can be obtained by solving a simple sub-optimization problem. The above effects ensure correctness of the important assumption $\frac{\|\delta s(\boldsymbol{x},t)\|}{\|s(\boldsymbol{x},t)\|} \ll 1$ of the adjoint method in a large range. Thus, we can expect a large convergence domain of the new earthquake location method. According to the numerical experiments, the range of convergence is significantly enlarged. We also remark that there have been many efforts in expanding the range of convergence for the inverse problem, see e.g. [8,10,24, 25,38]. Here we provide a simple and alternative implementation.

The paper is organized as follows. In Section 2, the conventional waveform based