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Asymptotic-Preserving Discrete Schemes for Non-Equilibrium Radiation Diffusion Problem in Spherical and Cylindrical Symmetrical Geometries

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Abstract. We study the asymptotic-preserving fully discrete schemes for non-equilibrium radiation diffusion problem in spherical and cylindrical symmetric geometry. The research is based on two-temperature models with Larsen's flux-limited diffusion operators. Finite volume spatially discrete schemes are developed to circumvent the singularity at the origin and the polar axis and assure local conservation. Asymmetric second order accurate spatial approximation is utilized instead of the traditional first order one for boundary flux-limiters to consummate the schemes with higher order global consistency errors. The harmonic average approach in spherical geometry is analyzed, and its second order accuracy is demonstrated. By formal analysis, we prove these schemes and their corresponding fully discrete schemes with implicitly balanced and linearly implicit time evolutions have first order asymptotic-preserving properties. By designing associated manufactured solutions and reference solutions, we verify the desired performance of the fully discrete schemes with numerical tests, which illustrates quantitatively they are first order asymptotic-preserving and basically second order accurate, hence competent for simulations of both equilibrium and non-equilibrium radiation diffusion problems.

AMS subject classifications: 65M08, 80M35, 65M12

Key words: Spherical symmetrical geometry, cylindrical symmetrical geometry, non-equilibrium radiation diffusion problem, fully discrete schemes, asymptotic-preserving, second order accuracy.

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1 Introduction

Non-equilibrium radiation diffusion problems often appear in inertial confinement fusion (ICF), astrophysical phenomena, combustion and other research fields. Two-

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temperature model is a representative non-equilibrium problem. In case thermal equilibration is much faster than radiation diffusion, the system is called to be in its equilibrium limit, and can be described by a one-temperature model, which is the asymptotic limit of the two-temperature model. When simulating non-equilibrium radiation diffusion problems using numerical method, asymptotic-preserving (AP) discrete schemes are favored since they are able to maintain this asymptotic property and appropriate for both non-equilibrium and equilibrium radiation diffusions.

To demonstrate the notion of AP scheme more vividly, people often start from an original problem with a scaling parameter $\varepsilon \in [0,1]$. In this application, ε may differ in several orders of magnitude from the non-equilibrium regime ($\varepsilon = 1$) to the equilibrium regime ($\varepsilon = 0$). The smaller ε is, the closer the system is to the equilibrium limit. A discrete scheme of the original problem is called to be asymptotic-preserving if it converges to a scheme consistent with the limit problem when ε tends to zero [1]. An AP scheme works uniformly with respect to the parameter ε, hence adapts to both the original problem and its limit problem.

There are extensive publications studying the AP schemes and properties for transport and hydrodynamic problems [1–7]. These methods are also extended to other fields. For instance, for P_1 equation, some AP finite volume schemes were established on unstructured meshes in [4]. For discrete-velocity kinetic equations, some high order AP DG schemes were developed in [8]. For a non-equilibrium radiative transfer system, the classic Marshak wave equation and a higher order equilibrium diffusion approach were studied in [9]. However, studies on AP schemes for radiation diffusion problems are rarely found, although their importance in radiation multi-physics is pointed out and some researches on schemes considering accuracy and fast solutions have been developed [10–14].

Benefited from the work for the implicitly balanced (IB) and linearly implicit (LI) time integrations with asymptotic analysis and qualitative numerical tests in [15], we developed some AP finite volume schemes for non-equilibrium radiation diffusion problem in plane geometry in [16]. Therein the sensitivity of the global accuracy to the boundary flux-limiter approaches is explored, and second order spatial accuracy (i.e., $\mathcal{O}(h^2)$ spatial accuracy where h is a nominal mesh size and as $h \to 0$) is gained. However, that paper does not involve the curve geometrical problems. In practice, many heat transfer and radiation hydrodynamic problems arise in complicated geometry instruments, which are more appropriately modeled by using curved geometric coordinate systems such as spherical and cylindrical coordinates for simulation convenience [17–22]. For example, in [18], a radiation hydrodynamics (RHD) code was developed in Cartesian, cylindrical, and spherical geometries to simulate ICF involved problems. In [22], an axis-symmetric three-temperature non-equilibrium RHD code was developed to simulate intense thermal radiation or high-power laser driven radiative shock hydrodynamics in cylindrical and spherical symmetric geometry. In this paper, motivated by these broad applications, we focus on developing AP fully discrete schemes with high accuracy for