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Dispersive Shallow Water Wave Modelling. Part IV: Numerical Simulation on a Globally Spherical Geometry

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Abstract. In the present manuscript we consider the problem of dispersive wave simulation on a rotating globally spherical geometry. In this Part IV we focus on numerical aspects while the model derivation was described in Part III. The algorithm we propose is based on the splitting approach. Namely, equations are decomposed on a uniform elliptic equation for the dispersive pressure component and a hyperbolic part of shallow water equations (on a sphere) with source terms. This algorithm is implemented as a two-step predictor-corrector scheme. On every step we solve separately elliptic and hyperbolic problems. Then, the performance of this algorithm is illustrated on model idealized situations with even bottom, where we estimate the influence of sphericity and rotation effects on dispersive wave propagation. The dispersive effects are quantified depending on the propagation distance over the sphere and on the linear extent of generation region. Finally, the numerical method is applied to a couple of real-world events. Namely, we undertake simulations of the BULGARIAN 2007 and CHILEAN 2010 tsunamis. Whenever the data is available, our computational results are confronted with real measurements.

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1 Introduction

Until recently, the modelling of long wave propagation on large scales has been performed in the framework of Nonlinear Shallow Water Equations (NSWE) implemented

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under various software packages [52]. This model is hydrostatic and non-dispersive [79]. Among popular packages we can mention, for example, the TUNAMI code [40] based on a conservative finite difference leap-frog scheme on real bathymetries. This code has been extensively used for tsunami wave modeling by various groups (see *e.g.* [91]). The code MOST uses the directional splitting approach [84, 85] and is also widely used for the simulation of tsunami wave propagation and run-up [82,88]. The MGC package [75] is based on a modified MACCORMACK finite difference scheme [24], which discretizes NSWE in spherical coordinates. Obviously, the MGC code can also work in CARTESIAN coordinates as well. This package was used to simulate the wave run-up on a real-world beach [37] and tsunami wave generation by underwater landslides [4]. Recently the VOLNA code was developed using modern second order finite volume schemes on unstructured grids [22]. Nowadays this code is essentially used for the quantification of uncertainties of the tsunami risk [3].

All numerical models described above assume the wave to be non-dispersive. However, in the presence of wave components with higher frequencies (or equivalently shorter wavelengths), the frequency dispersion effects may influence the wave propagation. Even in 1982 in [60] it was pointed out:

> [. . .] *the considerations and estimates for actual tsunamis indicate that nonlinearity and dispersion can appreciably affect the tsunami wave propagation at large distances.*

Later this conclusion was reasserted in [68] as well. The catastrophic Sumatra event in 2004 [80] along with subsequent events brought a lot of new data all around the globe and also from satellites [54]. The detailed analysis of this data allowed to understand better which models and algorithms should be applied at various stages of a tsunami life cycle to achieve the desired accuracy [12, 62]. The main conclusion can be summarized as follows: for a complete and satisfactory description of a tsunami wave life cycle on global scales, one has to use a nonlinear dispersive wave model with moving (in the generation region [18]) realistic bathymetry. For trans-oceanic tsunami propagation one has to include also Earth's sphericity and rotation effects. A whole class of suitable mathematical models combining all these features was presented in the previous Part III [43] of the present series of papers.

At the present time we have a rather limited amount of published research literature devoted to numerical issues of long wave propagation in a spherical ocean. In many works (see *e.g.* [27,39]) Earth's sphericity is not taken explicitly into account. Instead, the authors project Earth's surface (or at least a sub-region) on a tangent plane to Earth in some point and computations are then performed on a flat space using a BOUSSINESQ-type (Weakly Nonlinear and Weakly Dispersive — WNWD) model without taking into account the CORIOLIS force. We notice that some geometric defects are unavoidable in this approach. However, even in this simplified framework the importance of dispersive effects has been demonstrated by comparing the resulting wave field with hydrostatic (NSWE) computations.

In [57] the authors studied the transoceanic propagation of a hypothetical tsunami