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Substructuring Preconditioners with a Simple Coarse Space for 2-D 3-T Radiation Diffusion Equations

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Abstract. Inspired by [Q. Y. Hu, S. Shu and J. X. Wang, Math. Comput., 79 (272) (2010): 2059-2078], we firstly present two nonoverlapping domain decomposition (DD) preconditioners B_h^a and B_h^{sm} about the preserving-symmetry finite volume element (SFVE) scheme for solving two-dimensional three-temperature radiation diffusion equations with strongly discontinuous coefficients. It's worth mentioning that both B_h^a and B_h^{sm} involve a SFVE sub-system with respect to a simple coarse space and SFVE sub-systems which are self-similar to the original SFVE system but embarrassingly parallel. Next, the nearly optimal estimation $\mathcal{O}((1+\log\frac{d}{h})^3)$ on condition numbers is proved for the resulting preconditioned systems, where d and h respectively denote the maximum diameters in coarse and fine grids. Moreover, we present algebraic and parallel implementations of B_h^a and B_h^{sm}, develop parallel PCG solvers, and provide the numerical results validating the aforementioned theoretical estimations and stating the good algorithmic and parallel scalabilities.

AMS subject classifications: 65F10, 65F15, 65N55, 65Z05

Key words: 2-D 3-T radiation diffusion equations, nonoverlapping domain decomposition, simple coarse space, condition number, parallel scalability.

1 Introduction

Radiation diffusion equations appear in a wide range of applications such as astrophysics and inertial confinement fusion (ICF). Two-dimensional three-temperature (2-D 3-T) radiation diffusion equations are utilized to approximately describe the radiative transition process and energy exchanges among photons, electrons and ions in the case neither of

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radiation and material energies reaches thermodynamic equilibrium [1–3]. On account of strong discontinuity and high nonlinearity features in multicomponent mediums, a time-consuming portion (generally more than 80%) in ICF simulations is how to pursue approximate solutions of the discretized 2-D 3-T radiation diffusion equations, since they inherently suffer from ill-conditioned systems of linear equations. Preconditioned Krylov subspace techniques are the most favorable choices, but the selected preconditioner has substantial impacts on the robustness and computational efficiency. Numerous preconditioners have been constructed in recent years, see [4–11] and the references cited therein. It is worth noting that the existing preconditioners can be categorized into incomplete LU factorization [12], geometric and algebraic multigrid [13–16] and their symmetric/nonsymmetric combinations.

Nonoverlapping domain decomposition (DD) method, based on a decomposition of the spatial domain of the problem into several mutually disjoint subdomains, has been eagerly concerned and researched by a plurality of scholars with the primary motivations of the inherent parallelism and high-efficiency to handle the problems with large jumps in coefficients [17]. The construction of an appropriate coarse space is the key ingredient to form an efficient nonoverlapping DD preconditioner. Plenty of such preconditioners have been respectively put forward to different models for decades, such as the classical BPS-type [18–20], Neumann-type [21,22], FETI-type [23–25] and BDDC-type [26,27]. But there are still several drawbacks on feasibilities of these preconditioners, for example, sophisticated or much bigger coarse spaces are needed for certain complicated problems, or even in demand of some intricate extensions on coarse spaces. Nonoverlapping DD preconditioners with a simple coarse space, which was proposed in [28] and can be viewed as variants of BPS-type, have attracted considerable attentions because of much simpler coarse solvers and lower computational complexity. From the survey of references, fewer studies on nonoverlapping DD techniques have been made to the preserving-symmetry finite volume element (SFVE) discretization [29] of 2-D 3-T radiation diffusion equations.

In this paper, on the basis of [28], we develop two nonoverlapping DD preconditioners, one is an additive preconditioner B_h^a and the other is a semi-multiplicative preconditioner B_h^{sm}. Both of them consist of two different types of subsystems corresponding to the coarse triangulation $\mathcal{T}_d$ and subdivisions associated with each edge in the triangulation $\mathcal{T}_h$. It must be pointed out that inexact solvers can be used to the coarse subsystem and other subsystems are completely independent of one another. Under some assumptions, we acquire the nearly optimal estimation $\mathcal{O}((1+\log\frac{d}{h})^3)$ on condition numbers of these two preconditioned systems in the framework of the auxiliary space method. Moreover, their algebraic and parallel implementations have been produced to interface practical applications smoothly. The numerical results for a typical physical model show that their convergence rates are only weakly dependent on mesh sizes regarding a fixed coarse triangulation in conformity with our theoretical results, and their parallel PCG solvers are robust and possess favorable algorithmic and parallel scalabilities.

The rest of this paper proceeds as follows. We present 2-D 3-T radiation diffusion equations and the SFVE discretization, and revisit several auxiliary results in Section 2.