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On the Stability and Accuracy of the BGK, MRT and RLB Boltzmann Schemes for the Simulation of Turbulent Flows

Patrick Nathen[1,*], Daniel Gaudlitz[2], Mathias J. Krause[3] and
Nikolaus A. Adams[1]

[1] *Insitute of Aerodynamics and Fluid Mechanics, Technische Universität München, Boltzmannstrasse 15, 85748 Garching bei München, Germany.*
[2] *Institute of Automotive Engineering, University of Applied Sciences Zwickau, Scheffelstraße 39, 08066 Zwickau, Germany.*
[3] *Institute for Mechanical Process Engineering and Mechanics, Institute for Applied and Numerical Mathematics, Karlsruhe Institute of Technology, Strasse am Forum 8, 76131 Karlsruhe, Germany.*

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Abstract. This paper presents an analysis of the stability and accuracy of different Lattice Boltzmann schemes when employed for direct numerical simulations of turbulent flows. The Single-Relaxation-Time scheme of Bhatnagar, Gross and Krook (BGK), the Multi-Relaxation-Time scheme (MRT) and the Regularized Lattice Boltzmann scheme (RLB) are considered. The stability and accuracy properties of these schemes are investigated by computing three-dimensional Taylor-Green vortices representing homogeneous isotropic turbulent flows. Varying Reynolds numbers and grid resolutions were considered. As expected, the BGK scheme requires sufficiently high grid resolutions for stable and accurate simulations. Surprisingly, the MRT scheme when used without any turbulence model fails to obtain mesh convergence for the type of flow considered here. The RLB scheme allows for stable simulations but exhibits a strong dissipative behavior. A similar behavior was found when employing the mentioned LBM schemes for numerical simulations of turbulent channel flows at varying Reynolds numbers and resolutions. The obtained insights on accuracy and stability of the considered Lattice Boltzmann methods can become useful especially for the design of effective turbulence models to be used for high Reynolds number flows.

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[*] Corresponding author. *Email addresses:* `patrick.nathen@aer.mw.tum.de` (P. Nathen), `Daniel.Gaudlitz@aer.mw.tum.de` (D. Gaudlitz), `mathias.krause@kit.edu` (M. J. Krause), `nikolaus.adams@tum.de` (N. A. Adams)

1 Introduction

In the last three decades, the Lattice Boltzmann Method (LBM) became a promising alternative to conventional methods, like solving the Navier-Stokes equations with Finite Volumes or Finite Elements. Flows through porous media, multi-phase and multicomponent flows with and without heat transfer as well as flows around complex geometries have been investigated by several authors, see, e.g. [1,7]. With LBM the computational domain is discretized by an equidistant mesh, on which a discrete set of velocity distribution functions is solved numerically. This set of velocity distribution functions corresponds to discrete lattice velocities, which are used to recover the macroscopic moments in terms of a Hermite Polynomial expansion. Although the LBM has been applied to a wide range of fluid-dynamics applications, turbulence modeling in the LBM framework still requires considerable research and has not reached the level of maturity as seen for Navier-Stokes-based methods, see references [37,38] and [39]. Important recent advances have been made by Sagaut et al., see Sagaut [39] and Malaspinas and Sagaut [31, 32]. The approach proposed by Malaspinas and Sagaut [31] is based on the Approximate Deconvolution Method (ADM) of Stolz and Adams [40]. The discrete Boltzmann equations are filtered and subsequently deconvoluted with a regularized inverse filter operation in order to reconstruct the proper macroscopic equations for LES within the kinetic theory. The ADM as implemented in [31] is based on the BGK collision approach, yet it can be extended for any other standard collision operator like the MRT and the RLB approach. Due to the substantial differences of the different collision models, see also Section 2, it is crucial to understand their properties in terms of stability and accuracy, when used for the simulation of turbulent flows. Moreover, these properties have to be taken into account when adapting turbulence models to LBM schemes.

To the author's knowledge, a comparison of the stability and accuracy of different discrete LBM schemes without turbulence models when applied to three-dimensional turbulent flows of varying Reynolds number and employing different mesh resolutions has not been presented so far. For Navier-Stokes equation based methods it is common practice to investigate the properties of the discretization scheme in terms of spatial and temporal properties. The Lattice Boltzmann algorithm corresponds to a finite difference scheme with spatial accuracy $\mathcal{O}(\Delta x^2)$ and a temporal accuracy $\mathcal{O}(\Delta t)$. This work aims to investigate the stability and accuracy of different discrete collision schemes for the LBM by carrying out resolved and under-resolved Direct Numerical Simulations (DNS) of Homogeneous Isotropic Turbulence at varying Reynolds numbers. Since no turbulence models will be employed, the observed dissipation will be only due to viscous effects and numerical dissipation of the selected collision scheme. For selected LBM schemes we will provide insights on the accuracy and stability, which can be useful for the further development of turbulence models in the LES-LBM framework. The effectiveness of turbulence models strongly depends on the properties of the numerical scheme employed to solve the basic flow equations. For the type of flow considered in this work some of the recently presented LBM schemes can provide stable simulations of appropriate accuracy.