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# High-Order Vertex-Centered U-MUSCL Schemes for Turbulent Flows

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**Abstract.** Many production and commercial unstructured CFD codes provide no better than 2nd-order spatial accuracy. Unlike structured grid procedures where there is an implied structured connectivity between neighboring grid points, for unstructured grids, it is more difficult to compute higher derivatives due to a lack of explicit connectivity beyond the first neighboring cells. Our goal is to develop a modular high-order scheme with low dissipation flux difference splitting that can be integrated into existing CFD codes for use in improving the solution accuracy and to enable better prediction of complex physics and noise mechanisms and propagation. In a previous study, a 3rd-order U-MUSCL scheme using a successive differentiation method was derived and implemented in FUN3D. Verification studies of the acoustic benchmark problems showed that the new scheme can achieve up to 4th-order accuracy. Application of the high-order scheme to acoustic transport and transition-to-turbulence problems demonstrated that with just 10% overhead, the solution accuracy can be dramatically improved by as much as a factor of eight. This paper examines the accuracy of the high-order scheme for turbulent flow over single and tandem cylinders. Considerably better agreement with experimental data is observed when using the new 3rd-order U-MUSCL scheme.

**AMS subject classifications**: 76M12

**Key words**: High-order scheme, CFD, turbulent flow, FUN3D.

## 1 Introduction

Even though the unstructured grid methodology is gaining popularity in the CFD community with increasing emphasis on solving flow over more complex configurations, it is observed that almost all production unstructured CFD codes, such as FUN3D, USM3D, and Loci/CHEM, etc., and commercial CFD codes, such as ANSYS-Fluent, CFD-ACE+,

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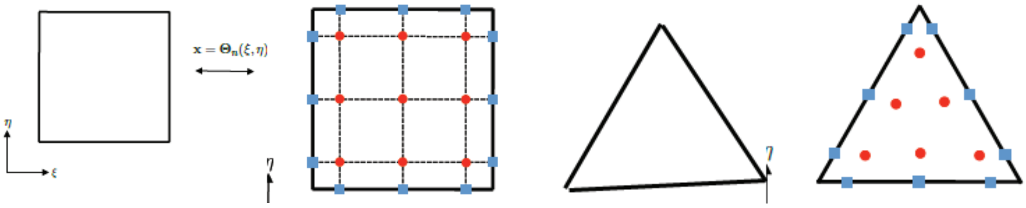

Figure 1: Example stencil for DG scheme for achieving high-order accuracy by adding extra DOFs within each cell.

STAR-CCM+, use 2nd-order spatial schemes. Unlike structured grid procedures where there is an implied structured connectivity between neighboring grid points, for unstructured grids, it is more difficult to compute higher derivatives due to a lack of explicit connectivity beyond the first neighboring cells.

There has been considerable interest and activities in high-order scheme development for unstructured grids using variants of the discontinuous Galerkin (DG) method. In the DG method, one introduces extra degrees-of-freedom (DOFs) in each cell, as shown in Fig. 1 to fit a high-order polynomial to the solution. In a sense, a ”structured” connectivity is recovered within each cell in the DG method.

Table 1 lists the number of required extra DOFs in each cell to achieve high-order accuracy when using the DG method. In order to keep the same total DOFs for a given problem, one must use a much coarser initial grid compared to that used by standard 2nd-order codes. The formulation of DG methods is also very different from the classical finite volume method in that all DOFs within a cell are tightly linked together, and without special treatment, the ”mass” matrix for that cell can be a full matrix (rather than a diagonal matrix), and must be stored and inverted implicitly. As such, the implementation of DG methods into existing production CFD codes requires substantial code modifications. Furthermore, the development of a new production code to take advantage of a new high-order scheme would be prohibitively expensive and require considerable verification and validation efforts.

Table 1: Extra DOFs for high-order schemes using DG.

| | $p=1$, 2nd order | $p=2$, 3rd order | $p=3$, 4th order |
|---|---|---|---|
| 2D Triangle, $(p+1)(p+2)/2$ | 3 | 6 | 10 |
| 2D Quadrilateral, $(p+1)^2$ | 4 | 9 | 16 |
| 3D Tetrahedral, $(p+1)(p+2)(p+3)/6$ | 4 | 10 | 20 |
| 3D Hexahedral, $(p+1)^3$ | 8 | 27 | 64 |

In this study, we develop an unstructured high-order scheme with low dissipation flux difference splitting that can be integrated into any production unstructured CFD code (such as FUN3D, USM3D, or Loci/CHEM) for noise prediction. The effort builds