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Joint Optimization of the Spatial and the Temporal Discretization Scheme for Accurate Computation of Acoustic Problems

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Abstract. Here, a physical dispersion relation preserving (DRP) scheme has been developed by combined optimization of the spatial and the multi-stage temporal discretization scheme to solve acoustics problems accurately. The coupled compact difference scheme (CCS) has been spectrally optimized $(OCCS)$ for accurate evaluation of the spatial derivative terms. Next, the combination of the $OCCS$ scheme and the five stage Runge-Kutta time integration $(ORK5)$ scheme has been optimized to reduce numerical diffusion and dispersion error significantly. Proposed $OCCS-ORK5$ scheme provides accurate solutions at considerably higher CFL number. In addition, $ORK5$ time integration scheme consists of low storage formulation and requires less memory as compared to the traditional Runge-Kutta schemes. Solutions of the model problems involving propagation, reflection and diffraction of acoustic waves have been obtained to demonstrate the accuracy of the developed scheme and its applicability to solve complex problems.

AMS subject classifications: 65N06, 65N35, 76D05, 35L05

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1 Introduction

Computational acoustics problems demand space-time accurate simulation of an acoustic field over a long duration. Such problems are solved by researchers using an acoustic

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analogy [1] as well as by solving linearized Euler equations [2, 3]. Simulations of acoustics problems require high accuracy scheme for evaluating various derivatives present in the linearized Euler equations. Acoustic waves consist of unsteady pressure fluctuations with amplitude several order smaller compared to the background atmospheric pressure. These fluctuations display a wide band phenomenon starting from few hundred to few kilo hertz of frequencies. It is important to numerically resolve all the spatial and temporal scales in the fluid flow. Propagation of acoustic waves in air over a small distance displays non-dispersive and non-dissipative nature [4, 5]. However, most of the numerical methods are not useful for solving such problems due to their dispersive and dissipative nature. Thus, development, analysis and use of new high accuracy methods is important while solving such class of problems.

Vichnevetsky [6] solved hyperbolic equations to analyze wave propagation phenomenon using finite difference approach. However, numerical dispersion analysis provided in [6] was wrong and has been later corrected in [5, 17]. Estimation of numerical phase and numerical group velocity properties to understand presence of spurious waves in the computed solution has been discussed in [7, 8]. It is not the order but the resolving ability of the numerical method that decides the accuracy of the computed solution [5, 9]. Two numerical methods can have same order of accuracy but completely different resolving abilities and the method with higher resolving ability provides more accurate results. In this regard, compact schemes are developed and widely used by different researchers for the discretization of spatial derivative terms since they offer high spectral resolution even with a relatively smaller stencil [10, 26].

Ekaterinaris [11] developed compact difference schemes and used them for solving hyperbolic equations corresponding to the gas dynamics and aeroacoustic test problems. Pradhan et al. [12] derived a coupled compact difference scheme (CCS) for the solution of aeroacoustic problems with an attractive feature of adaptive numerical diffusion. Resolving ability of a finite difference scheme can be further enhanced by optimization process as discussed in [13, 21–24]. For a non-periodic problem, one is forced to use different stencils at the boundary, near-boundary and the interior points. Sengupta et al. [9, 14] proposed a global spectral stability methodology to analyze central and upwind compact schemes for the estimation of the spectral resolution, numerical amplification and numerical group velocity properties of a individual grid point in a non-periodic domain.

In addition to the excellent spectral resolution for the spatial derivative terms, used numerical schemes must also ensure that the resolved scales in the computed solution propagate at the correct physical speed. Such schemes are classified as the DRP schemes [5]. The spatial and the temporal scales are linked to each other through the physical dispersion relation. Although numerical schemes solve the governing differential equations, corresponding numerical dispersion relation differs from the physical one across a complete or a band of wavenumber range due to numerical inaccuracies. The optimization of DRP schemes for solving computational acoustics problems was later followed in [21–24].

Improvements in the time integration scheme has been suggested in [15, 16] by con-