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A Reduced Basis Method for the Homogenized Reynolds Equation Applied to Textured Surfaces

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Abstract. In fluid film lubrication investigations, the homogenized Reynolds equation is used as an averaging model to deal with microstructures induced by rough or textured surfaces. The objective is a reduction of computation time compared to directly solving the original Reynolds equation which would require very fine computational grids. By solving cell problems on the microscale, homogenized coefficients are computed to set up a homogenized problem on the macroscale. For the latter, the discretization can be chosen much coarser than for the original Reynolds equation. However, the microscale cell problems depend on the macroscale film thickness and thus become parameter-dependent. This requires a large number of cell problems to be solved, contradicting the objective of accelerating simulations. A reduced basis method is proposed which significantly speeds up the solution of the cell problems and the computation of the homogenized coefficients without loss of accuracy. The suitability of both the homogenization technique and the combined homogenization/reduced basis method is documented for the application to textured journal bearings. For this purpose, numerical results are presented where deviations from direct solutions of the original Reynolds equation are investigated and the reduction of computational cost is measured.

AMS subject classifications: 35B27, 76D08, 76M50

Key words: Multiscale problems, homogenization, reduced basis method, Reynolds equation, lubrication, tribology.

1 Introduction

For the reduction of friction and wear of drive system components such as bearings, tribological optimization of surfaces is essential. Textured surfaces are reported to show potential for the improvement of hydrodynamic performance [1–3]. However, the optimal

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choice of geometry and distribution of the textures depending on a particular application is an open field of research with increasing interest [4]. For numerical simulations, the use of textured surfaces leads to the requirement of highly resolved computational grids when conducting direct simulations to adequately capture the microscale effects induced by the textures. Averaging techniques help to reduce the computational cost while still resolving the macroscale behavior. In case of homogenization, also information on the microscale behavior is provided. In combination with reduced basis methods, a further reduction of simulation time can be obtained.

The Reynolds equation [5] is used to describe fluid film lubrication between two smooth surfaces. For computations in the case of rough surfaces, an extended equation was developed by Patir and Cheng [6]. In their approach, which they call *average flow model*, shear and pressure flow factors characterize the differences between the flow behavior of rough and smooth surfaces. The mathematical concept of homogenization is another averaging technique, applied to the Reynolds equation or variants of it by, for instance, Jai [7], Buscaglia et al. [8] and Almqvist and Dasht [9]. The general idea of homogenization is the replacement of a multiscale partial differential equation (PDE) by a homogenized PDE reflecting the macroscale behavior of the original PDE. Almqvist et al. [10] showed that under certain symmetry conditions regarding the roughness the average flow model and the homogenization approach lead to the same results. They also state that the average flow model makes use of wrong boundary conditions if there is no roughness symmetry. Hence, they conclude that homogenization is the preferred method. Nonetheless, the average flow model or variants of it are widely used, usually regardless of roughness symmetry and without any certification concerning introduced errors. The homogenization technique on the other hand even provides another benefit: contradicting the statement in [4] that all averaging methods cause a loss of local flow information, it is easily possible to compute a higher-order approximation from the averaged solution with negligible additional computational cost. This higher-order solution approximates the Reynolds solution well and thus allows for capturing local pressure maxima. Due to the mentioned advantages, homogenization is our averaging method of choice.

Apart from the application to rough surfaces, the average flow model and the homogenization method can also be used in the case of textured surfaces, on which we concentrate in the following. The benefit of homogenization increases with increasing number of texture elements because direct simulations with the Reynolds equation would require finer and finer grids. However, the local cell problems (in this context sometimes also called micro-bearing problems) on the microscale, which have to be solved to obtain homogenized coefficients for assembling the homogenized PDE, are parameter-dependent: they contain the function describing the film thickness which depends on the global macroscale coordinates. Hence, for each grid point of a macroscale discretization, one cell problem has to be solved. This can make the homogenization method inefficient. One approach to overcome this problem is to use Taylor expansions for the computation of the homogenized coefficients, presented by Buscaglia and Jai [11, 12]. Only one