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A Hybrid Numerical Simulation of Supersonic Isotropic Turbulence

Luoqin Liu[1], Jianchun Wang[2], Yipeng Shi[1], Shiyi Chen[1,2] and X. T. He[1,3,*]

[1] *State Key Laboratory for Turbulence and Complex Systems, Center for Applied Physics and Technology, College of Engineering, Peking University, Beijing 100871, China.*
[2] *Department of Mechanics and Aerospace Engineering, Southern University of Science and Technology, Shenzhen 518055, China.*
[3] *Institute of Applied Physics and Computational Mathematics, Beijing 100088, China.*

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Abstract. This paper presents an extension work of the hybrid scheme proposed by Wang et al. [*J. Comput. Phys.* **229** (2010) 169-180] for numerical simulation of subsonic isotropic turbulence to supersonic turbulence regime. The scheme still utilizes an 8th-order compact scheme with built-in hyperviscosity for smooth regions and a 7th-order WENO scheme for highly compression regions, but now both in their conservation formulations and for the latter with the Roe type characteristic-wise reconstruction. To enhance the robustness of the WENO scheme without compromising its high-resolution and accuracy, the recursive-order-reduction procedure is adopted, where a new type of reconstruction-failure-detection criterion is constructed from the idea of positivity-preserving. In addition, a new form of cooling function is proposed, which is proved also to be positivity-preserving. With a combination of these techniques, the new scheme not only inherits the good properties of the original one but also extends largely the computable range of turbulent Mach number, which has been further confirmed by numerical results.

AMS subject classifications: 76F05, 76F50, 76F65

Key words: Supersonic turbulence, hybrid scheme, positivity-preserving ROR-WENO scheme, compact scheme.

1 Introduction

Compressible turbulence is of fundamental importance to a number of natural phenomena and industrial applications, including interstellar medium [1,2], solar winds [3], star-

*Corresponding author. *Email addresses:* lqliu@pku.edu.cn (L. Liu), wangjc@sustc.edu.cn (J. Wang), ypshi@coe.pku.edu.cn (Y. Shi), syc@pku.edu.cn (S. Chen), xthe@iapcm.ac.cn (X. T. He)

forming clouds in galaxies [4], high-temperature reactive flows [5], supersonic aircraft design [6] and inertial confinement fusion [7, 8]. With increasing computational resources, direct numerical simulations of incompressible turbulent flows have been routinely conducted for many canonical boundary conditions and geometries. Similar developments for compressible flows are desired in order to provide parameterizations needed for modeling complex compressible turbulence in relevant applications.

While the pseudo-spectral method for incompressible homogeneous isotropic turbulence in a periodic domain has been well established [9], such a standard method is no longer suitable for compressible turbulence at high Mach numbers due to the notorious Gibbs phenomenon [10]. This barrier can be overcome by either the shock-fitting approach [11] or the shock-capturing approach [12, 13]. Although the former guarantees more accurate representations of shocked flows, it is merely feasible in cases where the shock topology is extremely simple and no shock wave forms during the calculation. Since our present goal is to simulate compressible turbulence where shocklets form randomly, we discuss only the latter within the context of finite difference method (FDM), which can be mainly categorized into four classes, namely, the classical shock-capturing methods [13–20], the artificial viscosity methods [12, 21–24], the nonlinear filtering methods [25–28], and the hybrid methods [29–33]. For a review on these methods, the paper of [34] is highly recommended. At present paper, we only focus on the last one.

Briefly speaking, the hybrid methods are based on the idea of endowing a baseline spectral-like scheme with shock-capturing capability through local replacement with a classical shock-capturing scheme, where the shock sensor plays a key role. Along this direction, some progresses have been made in the past two decades. For example, Adams and Shariff [30] first considered a truly adaptive hybrid discretization, consisting of a baseline 5th-order compact upwind (CU) scheme coupled with a 5th-order essentially non-oscillatory (ENO) scheme, where the shock sensor is based on the local gradient of the flux vector components. Pirozzoli [31] expanded this method by transforming it into a fully conservative formulation, replacing the ENO scheme with weighted essentially non-oscillatory (WENO) scheme, and using the local density gradient as the shock sensor. This method was further improved by Ren et al. [32], who used the Roe type characteristic-wise reconstruction and introduced a complex weight function to gradually switch between CU scheme and WENO scheme at the interface. Zhou et al. [35] introduced a new family of CU scheme and combined this with WENO scheme. There are also several other studies that combine the usual, non-compact scheme and WENO scheme [36–38], but they have the very similar issues in shock detection and the interface treatments as discussed above.

Recently, Wang et al. [33] developed a novel hybrid scheme that is applicable to the numerical simulation of compressible isotropic turbulence with relatively high turbulent Mach number $M_t \lesssim 1.0$. This scheme utilizes a 7th-order WENO scheme for highly compression regions and an 8th-order compact central (CC) scheme for smooth regions, with the shock sensor being the shocklet detection algorithm given by Samtaney et al. [39]. In addition, a numerical hyperviscosity formulation is proposed to remove the alias er-