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Accelerating N-Body Simulation of Self-Gravitating Systems with Limited First-Order Post-Newtonian Approximation

Takayuki Tatekawa[1,2,*]

[1] *Department of Social Design Engineering, National Institute of Technology, Kochi College, 200-1 Monobe-Otsu, Nankoku, Kochi 783-8508, Japan.*
[2] *Research Institute of Science and Engineering, Waseda University, 3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan.*

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Abstract. In this study, an N-body simulation code was developed for self-gravitating systems with a limited first-order post-Newtonian approximation. The code was applied to a special case in which the system consists of one massive object and many low-mass objects. Therefore, the behavior of stars around the massive black hole could be analyzed. A graphics processing unit (GPU) was used to accelerate the code execution, and it could be accelerated by several tens of times compared to a single-core CPU for $N \simeq 10^4$ objects.

AMS subject classifications: 65Y05, 65Y10, 68U20, 83C10

Key words: Particle simulations, N-body simulations, general relativity, GPGPU.

1 Introduction

The formation of massive black holes is one of the most important problems in astrophysics. It was recently reported that a supermassive black hole (SMBH) exists at the center of the Milky Way [1, 2]. It has also been proposed that SMBHs, whose masses are estimated to be in the range of $10^6 - 10^{10} M_\odot$, can exist in other galaxies, according to the relation between the SMBH mass and the luminosity [3], or the SMBH mass and bulge mass [4]. In dwarf elliptical galaxies, the relation between massive black holes and nuclear stars has been discussed [5].

For globular clusters, the existence of massive black holes remains unclear. For example, observations have indicated that the globular cluster NGC-224-G1 (or Mayall II) orbiting M31 can possess a massive black hole [6, 7]. In another case, the existence of

*Corresponding author. *Email address:* `tatekawa@akane.waseda.jp` (T. Tatekawa)

a massive black hole in M15 (or NGC 7078) has also been discussed [8, 9]. However, definitive conclusions have not yet been reached.

Although several scenarios for the formation of SMBHs have been discussed [10], the exact scenario remains to be clearly identified. For example, if the seeds of SMBHs were stellar-mass black holes, there would be an insufficient amount of time for them to grow into such a massive black hole. The scenario where galaxies and SMBHs evolve together has also been discussed [11].

Gravity mainly affects the formation and evolution of astronomical objects. Regarding dynamical evolution, N-body simulations have always been performed. When we can see the effects of radiation or pressure of baryonic gas, we must consider the hydrodynamical evolution. In contrast, we consider only the gravitational interaction between objects where we can apply N-body simulations, in which the interactions are described by Newtonian gravity.

However, it is inadequate to describe the interaction of objects in neighboring regions of SMBHs by Newtonian gravity alone. In Newtonian gravity, Bahcall and Wolf demonstrated that when a globular cluster possesses a massive black hole, the density distribution peaks at the center of the cluster [12]. Although their work is quite important, when we discuss the behavior of stars near a massive black hole, the effect of general relativity becomes important. Therefore, we should consider the effect of general relativity in the neighboring regions of SMBHs. For the N-body simulation, a post-Newtonian (PN) approach was proposed. The interaction of massive objects is extended by $(v/c)^n$ terms. Then, the lowest-order term ($(v/c)^2$) is added to the Newtonian interaction. This equation of motion for N-body systems is known as the EIH equation [13, 14]. The equations of motion for N-body systems up to the second-order PN ($(v/c)^4$) have also been derived [15].

In this study, a numerical simulation code for PN N-body simulations was developed. Here, we note a special case, i.e., we suppose one SMBH and many stars. The interaction between the SMBH and stars are estimated by the PN approximation. Then, the interaction between stars is calculated by Newtonian gravity. In this case, the procedure of the computation is reduced. The interactions are computed on a graphics processing unit (GPU), which can process a large number of operations in parallel. Because the computation of complicated interactions is carried out on a GPU, the total computation time can be reduced.

The paper is organized as follows. In Section 2, the equation of motion and conserved quantities are described. In a generic case, because of the emission of gravitational waves, the total energy of the system decreases. In the first-order PN (1PN) approximation, because gravitational waves are not emitted, the total energy is conserved. Here, we note a special case, i.e., the system consists of one massive object and many low-mass stars. In Section 3, the numerical simulation is described. Using a GPU, the simulation can be accelerated. The elapsed time of the simulations are compared between cases of a central processing unit (CPU) only and a CPU+GPU. In Section 4, the time evolution for simple models is presented and the accuracy of the simulation is validated. Then, the time evo-