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Analysis of Local and Parallel Algorithm for Incompressible Magnetohydrodynamics Flows by Finite Element Iterative Method

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Abstract. Based on two-grid discretizations, a local and parallel finite element algorithm (LPFEA) based on Newton iteration for solving the stationary incompressible magnetohydrodynamics (MHD) is considered in this paper. The basic idea of the algorithm is to compute the nonlinear system by Newton iteration on a globally coarse mesh first, then solve a series of subproblems of residual correction on the corresponding subdomains with fine grids in parallel. The optimal error estimates with respective to iterative step m and mesh sizes H and $h \ll H$ are derived. The efficiency of the method is illustrated by numerical experiments.

AMS subject classifications: 35Q30, 65M60, 65N30, 76D05

Key words: Local and parallel algorithm, finite element, Newton iteration, stationary incompressible magnetohydrodynamics.

1 Introduction

The incompressible MHD model describes the interaction between a viscous, incompressible, electrically conducting fluid and an external magnetic field. The governing equations are a coupled system of Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism via Lorentz's force and Ohm's law. MHD is very important and widely used in technological and industrial applications, such as metallurgical engineering, electromagnetic pumping, stirring of liquid metals, and MHD generators, see [1–3]. There have been various numerical approaches about finite element

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method (FEM) for solving the MHD system, such as a finite element scheme based on regularized exact penalty formulation by Gunzburger et al. [4], mixed finite element method (MFEM) on general Lipschitz polyhedra by Schötzau [5], MFEM based on weighted regularization by Hasler et al. [6], unconditionally convergent stabilized finite element scheme by Badia et al. [7], two-level finite element iterations by Dong and He [8,9], a least-squares finite element formulation by Salah et al. [10]. As for the nonstationary incompressible MHD system, first-order and second-order implicit/explicit fully discrete finite element schemes are presented by Layton et al. [11], Euler semi-implicit fully discrete finite element schemes are analyzed by Prohl [12] and He [13]. Regarding the methodologies used for solving nonlinear partial differential equations, the two-level finite element algorithm [14–17] is one of the effective methods which can save much computing time and storage space. In addition, the multilevel iterative FEM studied in [18–20] can also treat the nonlinear problems efficiently.

Although computing power increases rapidly, constructing a highly refined, efficient algorithm for solving the incompressible MHD flows is still a big challenge. Based on the observation that the global behavior of a finite element solution is dominated by low frequencies and the local behavior is mostly governed by high frequencies, Xu and Zhou [21] designed a LPFEA to a class of elliptic boundary value problems. The LPFEA is thought as a higher performance method than classical Galerkin FEM, so it has been developed and applied to solve strong nonlinear systems, such as Navier-Stokes problem [22–24] and MHD equations [25] and so on. The algorithm is based on the two-grid technique and the domain decomposition method [26], in which the overlapping domain decomposition of the whole domain with the matching grids is used. Besides, for the convergence analysis of conforming FEM on overlapping nonmatching mesh based on the partition of unity method, we refer the interested readers to Huang and Xu [27].

So far, there are much less works on LPFEA for MHD problem. Inspired by [21, 28], as an extension of our recent work [25], the goal of this article is to design a LPFEA based on Newton iteration for the stationary incompressible MHD flows. On a globally coarse grid, the nonlinear system is solved by Newton iteration first, then the local subproblems of residual correction are computed in parallel on some subdomains with fine grids. The competitive superiorities of the algorithm are two aspects. It has low communication complexity and greatly reduces computing time, since the residual problems are independent of each other and only dependent of the coarse grid solution. The Newton iteration is exponentially convergent which is faster than other classical Stokes-type and Oseen-type iterations studied in [29].

This paper is arranged as follows. In the next section, we provide mathematical preliminaries and error bounds of MFEM for the steady incompressible MHD flows. In Section 3, based on the two-grid discretizations and overlapping domain decomposition technique, we propose the LPFEA based on Newton iteration, and analyze the error estimates with respect to the iterative step m and the mesh grid sizes H and h. In the last section, we carry out a series of numerical experiments to confirm the high efficiency of the proposed method.