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An Efficient Spectral Method for the Inextensible Immersed Interface in Incompressible Flows

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Abstract. In this paper, we study the equation system governing the movement of an immersed interface in incompressible fluid flows, and propose an efficient method for its numerical solution. The particularity of the current model is the inextensibility constraint imposed on the interface. We are interested in constructing a suitable variational formulation associated to this problem and the well-posedness of the weak problem. The significance of this variational formulation is that both the inextensibility of the interface and fluid incompressibility are strictly satisfied, and the well-posedness of the associated weak problem is rigorously proved. To the best of the authors' knowledge, no other models can be claimed to posses these properties. In fact our new formulation renders the inextensibility and the incompressibility constraints into a unique saddle point problem. Then, based on the proposed variational framework, we design an efficient spectral method for numerical approximations of the weak solution. The main contribution of this work are threefold: 1) a variational framework for the weak solutions of the immersed interface/incompressible equations and rigorous proof of the well-posedness of the weak problem; 2) a spectral method for solving the weak problem, together with a detailed stability analysis for the numerical solutions; 3) efficient implementation technique for the proposed method and some numerical experiments carried out to confirm the theoretical claims.

AMS subject classifications: 65M70, 74F10, 76N10

Key words: Spectral method, fluid-structure interaction, inextensible immersed interface, weak problem, well-posedness.

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1 Introduction

The fluid-interface interaction problem has applications in physics, biology, and material, including crystal growth, composite materials, multi-phase flows, cell deformation, micro-organism swimming, and so on. There are many methods to dealing with interface problems and each has its own good. A common approach in the literature is the so-called immersed boundary method (IBM), proposed by Peskin and applied by many others for simulating the blood flow and a number of related problems [1,3,5,19–21,23,24]. This approach takes also the name immersed interface method [12–15]. In the IBM the structure is viewed as being part of the surrounding fluid, thus only a single equation of fluid motion needs to be solved. On one side, the presence of the structure determines an Eulerian force distribution in the ambient fluid by the constitutive law of the object, which appears in the form of the Dirac delta function. On the other side, once the velocity is found, the position of the immersed structure is determined according to the local fluid velocity which is interpolated onto the immersed structure using the Dirac delta function.

Many problems in fluid-interface interactions involve deformable elastic bodies such as membranes or their two-dimensional simplified models, filaments, etc. In fact there are countless examples of dynamics involving the interactions between a fluid and a thin elastic sheet or a slender filament. For example, the problem of a flapping filament is of great interest due to its intriguing responses to the fluid flow, and has come to play an important role in understanding of animal locomotion in terms of its high propulsive efficiency and its capability of extracting energy from the underlying flows. An elastic thread is characterized by its bending and extension-compression elasticity (with an additional shear elasticity for an elastic membrane) [8]. A high extension-compression modulus makes the problem ill-conditioned, and thus makes the numerical solution difficult. In the case when the extension-compression modulus is very high, a common practice is to treat the thread or membrane as an inextensible interface or surface [4], leading to a constrained problem. Basically, it is about the Navier-Stokes equations with an Eulerian force distribution stemming from the immersed interface, subject to the incompressibility constraint in terms of the fluid velocity field:

$$\nabla \cdot \boldsymbol{u} = 0, \quad \text{in } \Omega,$$

and the inextensibility constraint on the interface:

$$\nabla_s \cdot \boldsymbol{u} = 0, \quad \text{on } \Gamma,$$

where Ω is the fluid domain, Γ is the interface immersed in Ω, and ∇_s is the surface gradient operator to be clarified. This is the model we are going to investigate in the present paper.

Although many studies in the literature have focused on such kind of inextensible interface problem; see, e.g., [9–11, 16, 18, 22, 25–27, 29], there are still some issues to be