Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0184

Vol. **25**, No. 4, pp. 1213-1234
April 2019

Generalized Multiscale Inversion for Heterogeneous Problems

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Received 31 August 2017; Accepted (in revised version) 2 February 2018

Abstract. In this work, we propose a generalized multiscale inversion algorithm for heterogeneous problems that aims at solving an inverse problem on a computational coarse grid. Previous inversion techniques for multiscale problems seek a coarse-grid medium properties, e.g., permeability and conductivity, and by doing so, they assume that there exists a homogenized representation of the underlying fine-scale permeability field on a coarse grid. Generally such assumptions do not hold for highly heterogeneous fields, e.g., fracture media or channelized fields, where the width of channels are very small compared to the scale of coarse grids. In these cases, grid refinement can lead to many degrees of freedom, and thus numerically unattractive to apply. The proposed algorithm is based on the Generalized Multiscale Finite Element Method (GMsFEM), which uses local spectral problems to identify non-localized features, i.e., channels (high-conductivity inclusions that connect the boundaries of the coarse-grid block). The inclusion of these features in the coarse space enables one to achieve a good accuracy. The approach is valid under the assumption that the solution can be well represented in a reduced-dimensional space spanned by multiscale basis functions. In practice, these basis functions are non-observable as we do not identify the fine-scale features of the permeability field. Our inversion algorithm finds the discretization parameters of the resulting system on the coarse grid. By doing so, we

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identify the appropriate coarse-grid parameters representing the permeability field instead of fine-grid permeability field. We illustrate the potential of the approach by numerical results for fractured media.

AMS subject classifications: 65M32, 65M60

Key words: Multiscale inversion, multiscale problem, generalized multiscale finite element method, coarse-grid.

1 Introduction

In many applications, one has to deal with medium properties of multiple scales and high contrast. For example, in subsurface applications, high-conductivity channels or fractures can appear in multiple locations and have complex geometries. Such features typically have multiple scales, e.g., very small widths and multiple (long) length scales. The related inverse problems include finding permeability (or channel distribution) from noisy and sparse pressure or concentration measurements, and they can be posed either as a regularized least squares formulation and/or within a Bayesian formulation.

There are several challenges when performing inversion using standard approaches (see the monographs [15, 25, 30, 38, 39] and references therein for details) for heterogeneous problems. Because of the presence of small scales, one needs to resolve multiple scales properly, which can lead to huge ill-posed systems that are difficult to solve. However, one cannot perform inversion on a coarse grid using standard approaches directly, since the latter implicitly assumes that there is a homogenized model (see, e.g., [16,18,36] for related inverse problems for homogenization). It was shown in [7, 11, 17] that this assumption is not valid for many practical multiscale problems, even at a low-order approximation. Indeed, because of the presence of high-contrast channels, one cannot use a single permeability or conductivity to represent a coarse-grid block. To remedy these drawbacks, multiple continuum approaches [2,3,31,37,40,41] can be employed instead; however, these approaches require multiple assumptions [8]. Meanwhile, using fine-grid discretizations can lead to many degrees of freedom without a priori knowledge of the locations of these thin features. In this paper, we present a novel generalized multiscale inversion algorithm, which employs our recent multiscale methods and solves inverse problem for discretization parameters rather than for fine-grid permeability fields. Thus by construction, it provides a low-dimensional inverse problem on the coarse grid and avoids many prior assumptions on the fine-grid geometry in order to regularize the inverse problem. The approach is in the spirit of regularization by discretization.

Next, we briefly discuss generalized multiscale methods in the context of inverse problems. We conceptually sketch it in Fig. 1, where we emphasize that one needs appropriate coarse-grid models (with multiple basis functions) for the inversion in order to achieve an accuracy within the error tolerance of the data. For simplicity, we consider a