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Locating Multiple Multipolar Acoustic Sources Using the Direct Sampling Method

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Abstract. This work is concerned with the inverse source problem of locating multiple multipolar sources from boundary measurements for the Helmholtz equation. We develop simple and effective sampling schemes for location acquisition of the sources with a single wavenumber. Our algorithms are based on some novel indicator functions whose indicating behaviors could be used to locate multiple multipolar sources. The inversion schemes are totally "*direct*" in the sense that only simple integral calculations are involved in evaluating the indicator functions. Rigorous mathematical justifications are provided and extensive numerical examples are presented to demonstrate the effectiveness, robustness and efficiency of the proposed methods.

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Key words: Inverse source problem, direct sampling method, Helmholtz equation, multipolar sources, acoustic wave.

1 Introduction

The inverse source problem concerns with the reconstruction of unknown sources from measured scattering data away from the sources. It arises in many scientific fields and engineering applications, such as antenna synthesis [6,17,40], active acoustic tomography [5,41], medical imaging [3,4,7,26] and pollution source tracing [22,27].

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Over recent years, intensive attention [6,8,9,17,23–25,37–40] has been focused on the inverse source problem of determining a source F in the Helmholtz equation

$$\Delta u+k^2u=F \quad \text{in } \Omega, \tag{1.1}$$

from boundary measurements $u|_\Gamma$ and $\partial_\nu u|_\Gamma$, where $k>0$ is the wavenumber, $\Omega\subset\mathbb{R}^N$ ($N=2,3$) is a bounded Lipschitz domain with boundary Γ and ν denotes the outward unit normal to Γ. A main difficulty of the inverse source problem with a single wavenumber is the non-uniqueness of the source due to the existence of non-radiating sources [3,10,12, 16,18], and several numerical methods with multi-frequency measurements [8,9,25,43,44] have been proposed to overcome it for the source with a compact support in the L^2 sense. However, fortunately, with a single wavenumber, the uniqueness can be obtained if *a priori* information on the source is available [19,24]. Readers may refer to [19–21,28] and the reference therein for the further stability results.

In this paper, we assume that the source F is a finite combination of well separated monopoles and dipoles of the form

$$F(x)=\sum_{j=1}^{M}(\lambda_j+\eta_j\cdot\nabla)\delta(x-z_j), \tag{1.2}$$

where δ stands for the Dirac distribution, $M\in\mathbb{N}$ signifies the number of the source points, $\{z_j\}_{j=1}^M$ are points in Ω, and λ_j and η_j are respectively, scalar and vector source intensities such that $|\lambda_j|+|\eta_j|\neq 0$ and $|\lambda_j\eta_j|=0, j=1,\cdots,M$. Here, the points $\{z_j\}_{j=1}^M$ are assumed to be mutually distinct. The inverse source problem under concern is a location acquisition problem, which can be stated as: Find the locations $\{z_j\}_{j=1}^M$ of the source F of the form (1.2) in the Helmholtz equation (1.1) from the boundary measurements $u|_\Gamma$ and $\partial_\nu u|_\Gamma$ with a single wavenumber k.

Numerical methods for determining the multipolar sources have drawn a lot of interest in the literature. For the Poisson equation, an algebraic method for recovering monopolar sources ($|\eta_j|=0$) was proposed in [23], and has been extended to the case of multipolar sources in [13,14,35,36]. The algebraic method has also been developed to the 3D Helmholtz equation in [24] for monopolar sources, and then to the 3D elliptic equations in [2] for multipolar sources. In the case of 2D elliptic equations, the method has been extended in [1] for monopolar sources. We refer to [29] for a relevant paper on the reconstruction of extended sources for the 2D Helmholtz equation.

The purpose of this paper is to provide simple and effective numerical methods for determining the multipolar sources for the Helmholtz equation in two and three dimensions. We focus our attention on the non-iterative sampling-type methods, and we shall develop some novel direct sampling schemes in this paper. The main technicality lies in the decaying property of oscillatory integrals, which is employed to construct indicator functions at any sampling point $z\in\Omega$ such that the proposed indicator functions attain the local maximum at z_j in an open neighborhood of z_j. We would like to emphasize that