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Neutron Discrete Velocity Boltzmann Equation and its Finite Volume Lattice Boltzmann Scheme

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Abstract. Simulation of neutron transport process plays an important role in nuclear reactor computation and the numerical technique becomes the focus of nuclear reactor engineering. This paper provides a neutron finite volume lattice Boltzmann method (NFV-LBM) for solving the neutron discrete velocity Boltzmann equation (NDVBE), in which the NDVBE is deduced from the neutron transport equation (NTE) and the NFV-LBM is obtained by integrating the NDVBE. The macroscopic conservation equations recovered from the NDVBE via multi-scale expansion shows that the NDVBE has higher-order accuracy than diffusion theory, and the numerical solutions of neutron transport problems reveal the flexibility and applicability of NFV-LBM. This paper may provide some alternative perspectives for solving the NTE and some new ideas for researching the relationship between the NTE and other approximations.

AMS subject classifications: 60-08, 60J60, 62E17, 65C20

Key words: Neutron transport, neutron discrete velocity Boltzmann equation, neutron finite volume lattice Boltzmann, diffusion theory.

1 Introduction

Study of neutron particle transport in a scattering and absorbing medium represents a kernel in nuclear reactor physics and can be governed by the neutron transport equation (NTE), whose mono-energy scheme can be written as [1]

$$\begin{aligned}&\frac{1}{v}\frac{\partial\psi(\mathbf{r},\mathbf{\Omega},t)}{\partial t}+\mathbf{\Omega}\cdot\nabla\psi(\mathbf{r},\mathbf{\Omega},\ t)+\Sigma_t(\mathbf{r},t)\psi(\mathbf{r},\mathbf{\Omega},t)\\&=\int_{4\pi}\Sigma_s(\mathbf{r})f(\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega})\psi(\mathbf{r},\mathbf{\Omega}',t)d\mathbf{\Omega}'+Q(\mathbf{r},\mathbf{\Omega},t),\end{aligned}\tag{1.1}$$

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where v is the speed of neutron particle; $\Omega = \boldsymbol{i}\mu + \boldsymbol{j}\eta + \boldsymbol{k}\xi = \boldsymbol{i}\sin\theta\cos\varphi + \boldsymbol{j}\sin\theta\sin\varphi + \boldsymbol{k}\cos\theta$ is the neutron transport direction with θ, φ represent the polar and azimuth angle, respectively; $\psi(\boldsymbol{r},\Omega,t)$ is neutron angular flux corresponding to position $\boldsymbol{r}$, angular direction Ω and time t; Σ_t and Σ_s are macroscopic total cross section and scattering cross section, respectively; $f(\boldsymbol{r},\Omega' \to \Omega)$ is the scattering phase function from direction Ω' to direction Ω; $Q(\boldsymbol{r},\Omega,t)$ is the neutron source.

Eq. (1.1) describes the mean neutron particle propagation within a given system [2], and has widely attracted the attention of various scientific communities, including neutron transport physics [1], nuclear reactor design [3], neutron capture therapies [4] and others [5, 6]. Due to its high dimensionality, simulation of this problem can be very difficult, especially for analytical methods. Thus, with the development of digital computer and numerical computing method, numerical techniques for solving this problem have received a substantial amount of interests from the research community.

In solving the neutron transport problem, a large number of techniques have been raised and developed based on the statistical approach, such as Monte Carlo method (MCM) [7–9], and deterministic techniques, including those of discrete-ordinates method (DOM, also be known as S_N) [10, 11], spherical harmonics method (P_N) [12], the method of characteristics (MOC) [13, 14] and finite different method (FDM) [15]. MCM is valuable as its high precision and capability of treating very complex configurations, but limited by its highly computational resource and stochastic statistical uncertainty [16, 17]. Thus, the deterministic methods, based on the solution of mathematical physics equations, have covered a wide scope of numerical methods and technologies. By using the DOM, the continuous solid angle domain can be discretized and the neutron angular flux is assumed constant within each direction, which leads to a series of discrete-ordinate NTEs [6]

$$\frac{1}{v}\frac{\partial \psi_\alpha(\boldsymbol{r},t)}{\partial t} + \Omega_\alpha \cdot \nabla \psi_\alpha(\boldsymbol{r},t) + \Sigma_t(\boldsymbol{r},t)\psi_\alpha(\boldsymbol{r},t) = S_{d,\alpha}(\boldsymbol{r},t) + Q_\alpha(\boldsymbol{r},t), \tag{1.2}$$

where the subscript α represents the index of neutron transport discrete direction and the neutron distribution function is defined as $\psi_\alpha(\boldsymbol{r},t) = \psi(\boldsymbol{r},\Omega,t)$; S_d is the scattering source term defined by

$$S_d(\boldsymbol{r},\Omega,t) = \Sigma_t(\boldsymbol{r},t) \sum_{\alpha'=1}^{M} \Delta\Omega_{\alpha'} \psi_{\alpha'}(\boldsymbol{r},t) f(\alpha,\alpha'). \tag{1.3}$$

Then, this series of NTEs can be solved by using conventional method such as MOC and FDM. It can be easily to find out that the number of computing equations is equal to the product of nodes number and directions number, which leads to the issue that when solving the multi-dimensional problem, the number of solving equations can easily exceed millions. Besides, the existing deterministic methods are relatively complicated for complex geometry. To improve these conditions, two types of thoughts may be considered: a novel technique for effectively resolving the NTE, and a further reduction for angular dependence.