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# An Implicit Discrete Unified Gas-Kinetic Scheme for Simulations of Steady Flow in All Flow Regimes

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**Abstract.** This paper presents an implicit discrete unified gas-kinetic scheme (DUGKS) for steady state flow simulation in all flow regimes. The DUGKS is a multi-scale finite volume method (FVM), which is able to recover accurately the Navier-Stokes solutions in the continuum regime and the free molecular transport in collisionless regime. In the transition regime, the DUGKS can present reliable solution as well due to the close coupling of particle transport and collision in the flux evaluation at a cell interface. In this paper, an implicit DUGKS is constructed with predicted iterative steps for the updating of macroscopic flow variables, then the updating of microscopic gas distribution function in a discrete velocity space. The lower-upper symmetric Gauss-Seidel (LU-SGS) factorization is applied to solve the implicit equations. The fast convergence of implicit discrete unified gas-kinetic scheme (IDUGKS) can be achieved through the adoption of a numerical time step with a large CFL number. Some numerical test cases, including the Couette flow, the lid-driven cavity flows under different Knudsen numbers and the hypersonic flow in transition flow regime around a circular cylinder, have been performed to validate the proposed IDUGKS. The computational efficiency of the IDUGKS for steady state flow computations in all flow regimes can be improved by one or two orders of magnitude in comparison with the explicit DUGKS.

**AMS subject classifications**: 76M12, 82C40, 76P05, 76K05

**Key words**: Implicit scheme, discrete unified gas-kinetic scheme, LU-SGS, all flow regimes.

## 1 Introduction

The simulation of multiscale flows over a wide range of Knudsen numbers becomes challenging for a numerical algorithm. Different flow physics with a large variation of temporal and spatial scales needs to be modelled and computed efficiently. In the scale on the particle mean free path, the flow dynamics cannot be modeled anymore as continuum hydrodynamics [1]. The particle based method, such as the direct simulation Monte

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Carlo (DSMC), needs to be adopted. But in continuum flow limit, the computational cost from particle method becomes unaffordable due to the constraint on the cell size and time steps [2].

The first kinetic scheme, which could give accurate Navier-Stokes solutions in all Reynolds number, is the gas-kinetic scheme (GKS) proposed by Xu [3]. The kinetic effect appears mostly in the discontinuous region, where the initial non-equilibrium distribution function and the particle transport provides a delicate dissipative mechanism for stabilization of the numerical shock structure. The GKS becomes more robust than many other Navier-Stokes solvers due to its un-split treatment of inviscid and viscous terms [4]. The time accurate solution used for the interface flux evaluation gives GKS more flexility and advantages in the development of higher-order schemes [5, 6]. In order to improve the efficiency of kinetic schemes, several studies have been done on the construction of implicit GKS. Chit et al. [7] applied approximate factorization and alternating direction-implicit (AF-ADI) method. With a large Courant-Friedrichs-Levy (CFL) number, a fast convergence is achieved in simulation of the inviscid compressible flows on structured grid. In comparison with the explicit GKS, this implicit method even obtains solutions with a better accuracy due to its implicit treatment. Li et al. [8] proposed an implicit GKS based on matrix free Lower-Upper Symmetric Gauss Seidel (LU-SGS) time marching scheme for simulation of hypersonic inviscid flows on unstructured mesh. Under unstructured refined meshes, implicit GKS is also proposed for turbulence simulation [9]. These methods can only be easily implemented on a hybrid unstructured mesh and good robustness is achieved. However, the problem for the GKS is that it can be used in the study of continuum flow.

For non-equilibrium flow computations, many kinetic schemes, such as the discrete ordinate method (DOM), have been constructed for solutions in the kinetic regime. But, most of them can be hardly used to simulate continuum flow efficiently because of the constraints on the time step and cell size to the order of particle collision time and mean free path [2]. Many other asymptotic preserving (AP) kinetic schemes successfully extend their schemes to the continuum flow, but for the inviscid Euler solutions only. For NS solutions, they cannot capture mass and momentum transport properly in boundary layer once the cell size and time step are much larger than the particle mean free path and collision time [10]. All these weakness is due to the decoupled treatment of particle transport and collision in the flux evaluation. In terms of multiscale flow simulation, due to the direct modeling on the cell size and time step the unified gas kinetic scheme (UGKS) makes a breakthrough and becomes the first finite volume method for the accurate simulation from the hydrodynamic scale Navier-Stokes solution to kinetic scale Boltzmann solution [2], which has been developed rapidly [11–26]. Based on the Boltzmann BGK model [27], the integral solution used in UGKS for the flux evaluation couples particle transport and collision, and the final solution for representing the flow regime depends on the ratio of the time step over the local particle collision time. The numerical time step isn't limited by the particle collision time in UGKS. Besides the update of gas distribution function, the UGKS updates the macroscopic flow variables as