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Error Analysis of the Plane Wave Discontinuous Galerkin Method for Maxwell's Equations in Anisotropic Media

Long Yuan[1] and Qiya Hu[2,3,*]

[1] *College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China.*
[2] *LSEC, Institute of Computational Mathematics and Scientic/Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.*
[3] *School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China.*

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Abstract. In this paper we investigate the plane wave discontinuous Galerkin method for three-dimensional anisotropic time-harmonic Maxwell's equations with diagonal matrix coefficients. By introducing suitable transformations, we define new plane wave basis functions and derive error estimates of the approximate solutions generated by the proposed discretization method for the considered homogeneous equations. In the error estimates, some dependence of the error bounds on the condition number of the coefficient matrix is explicitly given. Combined with local spectral element method, we further prove a convergence result for the nonhomogeneous case. Numerical results verify the validity of the theoretical results, and indicate that the resulting approximate solutions generated by the PWDG possess high accuracies.

AMS subject classifications: 65N30, 65N55

Key words: Time-harmonic Maxwell's equations, anisotropic media, plane-wave basis, error estimates, nonhomogeneous.

1 Introduction

The plane-wave method falls into the class of Trefftz methods [27], which has recently been systematically surveyed in [9]. The plane-wave method was first introduced to solve Helmholtz equation and was then extended to solve time-harmonic Maxwell's equations and elastic wave equations. Examples of this approach include the variational

*Corresponding author. *Email addresses:* `yuanlong@lsec.cc.ac.cn` (L. Yuan), `hqy@lsec.cc.ac.cn` (Q. Hu)

theory of complex rays (VTCR) [23, 24], the ultra weak variational formulation (UWVF) [2–5, 14–17, 31, 33], the plane-wave discontinuous Galerkin (PWDG) method [6–8, 30], the plane-wave least-squares (PWLS) method [10, 11, 22, 29, 32, 34] and the plane-wave least-squares combined with local spectral finite element (PWLS-LSFE) method [12,13]. Today, there are other dedicated computational strategies for the resolution of wave equations (see [1, 19, 28]).

Recently, the UWVF method was extended to solve homogeneous Maxwell's equations in anisotropic media [15]. The studies in [15] were devoted to approximating the Robin-type trace of the electric and magnetic fields in an anisotropic medium, and focus on the numerical tests and convergence analysis in TM mode scattering, which can result in a Helmholtz equation in two dimensions with an anisotropic coefficient.

For the three-dimensional anisotropic time-harmonic Maxwell's system, it was pointed out in [15, p.351] that *almost all theoretical questions related to the 3D UWVF approach to anisotropic media are still open: in particular, the revelent approximation properties of sums of anisotropic plane waves are not known* . To our knowledge, the other plane wave methods have not been generalized to solve the three-dimensional anisotropic Maxwell's equations. The existing numerical results indicate that the PWDG method can generate approximate solutions with higher accuracies.

In this paper we are mainly interested in extension of the PWDG method to the three-dimensional time-harmonic Maxwell's equations in anisotropic media (with diagonal matrix coefficients). Motivated by the results in [25], the permittivity ε and the permeability μ are assumed to be of the form

$$\begin{aligned} &\varepsilon=\varepsilon_r \mathrm{diag}(a_x,a_y,a_z)=\varepsilon_r\ \Lambda,\\ &\mu=\mu_r \mathrm{diag}(a_x,a_y,a_z)=\mu_r\ \Lambda, \end{aligned} \tag{1.1}$$

where the diagonal matrix $\Lambda=\mathrm{diag}(a_x,a_y,a_z)$, $\varepsilon_r,\mu_r,a_x,a_y$ and a_z are constant. Example electromagnetic problems within this class include the design of waveguides and antennas, scattering of electromagnetic waves from automobiles and aircraft, and the penetration and absorption of electromagnetic waves by dielectric objects. Under this assumption, a permittivity, ε, and permeability, μ, tensor can describe a linear metamaterial with no magnetoelectric coupling, where Bianisotropy effects have typically played a minor role in the overall response of the experimental metamaterials, and can be mitigated by design (see [18]).

In this paper, we propose a scaling transformation and a coordinate transformation and verify some stabilities of the two transformations. Based on these, we define new plane wave basis functions associated with the considered model and derive error estimates of the approximate solutions generated by the PWDG for the underlying Maxwell's equations. Moreover, by combining the ideas in [12] we apply the proposed method to solve anisotropic nonhomogeneous Maxwell's equations and derive the corresponding error estimates. The theoretical results show that the errors of the resulting approximate solutions are affected by the condition number of the anisotropic coefficient matrix