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The Finite Point Method for Solving the 2-D 3-T Diffusion Equations

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Abstract. A new approach for numerically solving 3-T diffusion equations on 2-D scattered point distributions is developed by the finite point method. In this paper, a new method for selecting neighboring points is designed, which is robust and well reflects variations of gradients of physical quantities. Based on this, a new discretization method is proposed for the diffusion operator, which results in a new scheme with the stencil of minimal size for numerically solving nonlinear diffusion equations. Distinguished from most of meshless methods often involving dozens of neighboring points, this method needs only five neighbors of the point under consideration. Numerical simulations show the good performance of the proposed methodology.

AMS subject classifications: 65D25, 65M06, 65M70

Key words: Finite point method, 2-D 3-T diffusion equations, minimal stencil, method for selecting neighboring points.

1 Introduction

Solving 3-T radiation diffusion equations is a very important topic in research of astrophysical phenomena and inertial confinement fusion problems. The equations describe heat diffusion and exchange of energy among electrons, ions and photons. They are highly nonlinear and strongly coupled. Moreover, in many applications, as the diffusion problems are often coupled with the hydrodynamics problems, which are often solved by using the Lagrangian method or the arbitrary Lagrangian Eulerian method, diffusion problems have always to be solved on distorted meshes. These features make it more challenging to solve diffusion equations, hence it is crucial to design efficient algorithms. In this fields, some mesh-based works have been developed (see [1–5] and references therein).

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In recent years, to avoid the difficulties in mesh generation for complex problems, meshless methods have received intensive attention (see [6–8] and references therein), and some of them have been applied to solve diffusion problems, such as meshfree finite difference methods [9, 10], radial basis function methods (RBFs) [11, 12], the finite point method [13, 14], and the meshfree Galerkin method [15]. These works mainly focus on a single diffusion equation, rather than on diffusion equation systems, and most of them consider discretization on the uniform point distribution.

In the present paper, we consider to design an efficient discretization method to solve 3-T equations on 2-D scattered point distributions by the finite point method (FPM). The FPM is a meshless method defined on the scattered point distributions, which is first proposed by [16] based on directional differences. It distinguishes from the FPM proposed by [13] in that it employs just adequate points to solve for derivatives in terms of given order, while the FPM in [13] employs much more points than unknowns resulting an overdetermined system solved by the weighted least-square procedure. Here and below, unless otherwise indicated, we always refer the FPM presented in [16]. The FPM is essentially an approach to solve PDEs by the finite difference method on scattered points. As a special case, the classical finite difference method is a meshfree method in nature, since it is only related to the uniform point distribution and no integral over the meshes is involved. However, it is difficult to generalize the classical finite difference method to the scattered points. As compared with the FEM and the FVM, without limitation of the mesh structure, the FPM often employs the more flexible neighborhood criteria, hence it is more apt to tackle problems with large deformation background, such as the 2-D 3-T problems. In the FPM regime, [16, 17] have successfully solved the diffusion equation on 2-D scattered point distributions, and both of them obtain numerical solutions with good accuracy.

The main features of the newly proposed approach are as follows: a well-designed method for selecting neighboring points is robust enough to deal with problems with large gradient, the discrete schemes of diffusion problems have minimal stencils, and the coefficients of the resulted scheme are explicitly given avoiding solving matrix equations.

The rest of the paper proceeds as follows: Section 2 describes the 2-D 3-T diffusion equations; Section 3 presents some preliminaries for the FPM; Section 4 formulates the discretization methodology including designing a new method for selecting steady distribution neighboring point set on scattered point distributions, and constructing a discrete scheme with minimal stencils for solving 2-D 3-T problems; Section 5 performs numerical simulations to demonstrate the good performance of the integrated algorithm; finally, some concluding remarks are made in Section 6.

2 2-D 3-T diffusion equations

Two-dimensional diffusion equations coupled with electron, ion and photon are defined as