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Direct Imaging of Inhomogeneous Obstacles in a Three-Layered Ocean Waveguide

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Abstract. In this work, we have proven the inhomogeneous obstacle can be uniquely determined by the measured acoustic pressure data and have proposed an extended multilevel sampling method. The extended recovery approach only applies the matrix-vector operations to estimate the inhomogeneous media from the received partial data. In practice, the method is capable of reconstructing the objects of different shapes and locations, robust against noise, computationally fairly cheap and easy to carry out. We can consider it as a simple and direct algorithm to supply satisfactory initial locations for the application of any existing more refined and precise but computationally more demanding techniques to achieve accurate reconstructions of physical features of scatterers.

AMS subject classifications: 35R30, 41A27, 76Q05

Key words: Three-layered ocean waveguide, direct recovery, multilevel sampling method, penetrable scatterers.

1 Introduction

The forward and inverse scattering problems of submerged acoustics have wide applications in identification of wreckages, navigational scatterers, submarines, mineral deposits and so on, and have received great attention in recent years; see [5, 6, 9–11, 17, 20, 28, 38] and references therein. One of the popular models applied for acoustics in a finite depth ocean is the one-layered waveguide bounded by two parallel planes, see the theoretical and numerical results in [1, 3, 14, 19, 22, 25, 33, 35].

It is well known that a typical sound velocity profile consists of the surface channel, the thermocline and isothermal layers [25]. In order to maintain the physical features of

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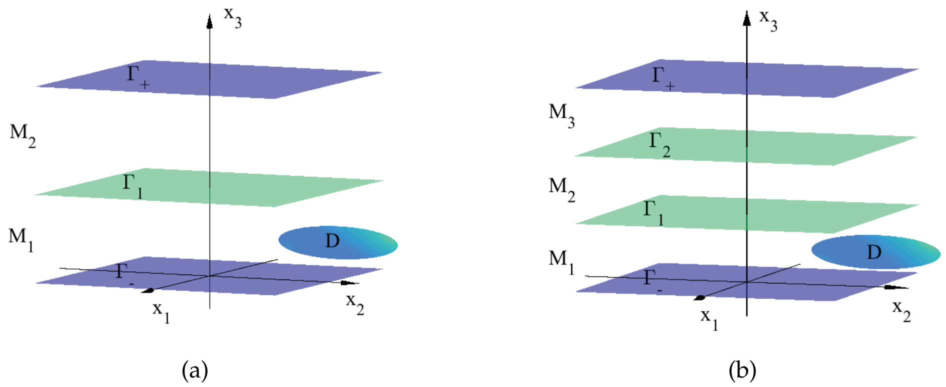

Figure 1: The demonstration of (a) two-layered waveguide and (b) three-layered waveguide.

ocean environment, the horizontal stratified waveguide in [2,7] is introduced as a simple but reasonably realistic model. In this model, we suppose the sound speed c depends essentially on the depth of the waveguide. A distinctive property of the submerged acoustic model is the transmission loss (propagation loss), which is because of the absorption of acoustic energy by the geometric spreading (divergence effect) and the propagation medium [7,32]. In addition, because of the reflection and refraction of the surface, bottom and the interfaces of each two adjacent layers, sound wave propagates in a special way. The Green's function needs to have such property, and it has been verified in [24]. Particularly, due to the presence of two parallel infinite boundaries of the waveguide, only a finite number of wave modes are able to propagate at a long distance, while the other modes decay exponentially [33]. All the above phenomenons increase the ill-posedness of the inverse problem notably, therefore the reconstruction of wave-penetrable scatterers in the stratified system is much more complex than that in the homogeneous one. Recently, most of the results are obtained for the two-layered waveguide model in Fig. 1(a), please refer to [13,16,18,27,34]. However, the three-layered model in Fig. 1(b) is more reasonable and practicable than the two-layered one, and very few studies [23, 24] are available in the literature for it, especially in the three dimensions. Consequently, we will investigate the three-layered ocean waveguide of the three dimensions and provide a uniqueness theorem and an extended multilevel sampling method for the inverse problem in this paper.

The novel uniqueness theorem shows that the embedded inhomogeneous medium scatterer can be uniquely determined by the same measured acoustic pressure data. And the uniqueness estimate of unknown objects by the proposed multilevel sampling method can be ensured by the theorem as well. The initial key step of the inverse scattering problem is to effectively approximate some regions that contain all the unknown objects. Otherwise, an extremely large computational domain brings generally in a huge additional computational burden for the entire reconstruction procedure when the initial key step is ignored. Our newly proposed multilevel sampling method (MSM) is devoted to