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An Adaptive High Order WENO Solver for Conservation Laws

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Abstract. This paper presents an implementation of the adaptive hybrid WENO (weighted essentially non-oscillatory) scheme based on our previous investigations for compressible multi-medium flows (Liu and Hu, J. Comput. Phys., 342 (2017), 43-65). In this study a simple and efficient method is developed for Euler equations and Navier-Stokes equations arising from the conservation laws. A class of high order weighted essentially non-oscillatory (WENO) schemes are applied to resolve the complicated flow structures and shock waves. Classical WENO schemes are computationally expensive in calculating the non-linear weight and smoothness indicators. We propose a block-structured adaptive mesh method together with a modified hybrid-WENO scheme to reduce the cost, the reconstruction is only performed at non-smooth region. Comparisons of WENO scheme with various smoothness indicators and different Lax-Friedrich flux vector splitting methods are performed on block structured adaptive mesh. Benchmark tests show present adaptive hybrid WENO method is low-dissipative and highly robust. The 2-D/3-D shock wave boundary layer interaction are simulated to verify the efficiency of present AMR (adaptive mesh refinement) solver in predicting turbulent flow.

AMS subject classifications: 76L05, 76F65

Key words: Block-structured AMR, WENO, Euler equations, Navier-Stokes equations, large scale computation.

1 Introduction

Accurate and efficient simulations of the hyperbolic conservation laws are of great importance in many applications including astrophysics, aerospace and weather forecast.

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A typical example is the compressible turbulent flows in aerodynamics, in which the numerical scheme should resolve smooth but complicated flow structures accurately and prevent spurious oscillations near shocks and shocklets. To satisfy the above properties, high resolution schemes are developed, like WCNS (weighted compact nonlinear schemes) [1] and the WENO scheme [2]. WENO schemes compute numerical fluxes by convex combination of several candidate stencils, with the final flux being approximated by summing weighted contributions of each stencil [3]. With the smoothness measurements, the flow discontinuity will produce a weight of almost zero to any stencil containing it. In smooth regions, the weights are designed to be optimal by which the maximum order of accuracy is obtained. Since the weights are highly dependent on the smoothness measurements, defining an effective smoothness indicator becomes an essential research topic. A well-documented comparison between WENO and other high order schemes is given by Brehm et al. [4].

Jiang and Shu [5] developed a robust implementation of a high order accurate scheme WENO_JS. Unfortunately, it often generates excessive numerical dissipation and may lose accuracy in the critical points. To fix the problem, WENO_M scheme [6] was proposed by using a mapping function to correct the weights. Although significant improvements have been made on the smooth and high gradients regions, the added mapping procedure is computationally expensive. A new WENO scheme, the WENO_Z scheme [7], in which new nonlinear weights are constructed by using the smoothness indicators of WENO_JS [5] and a new global smoothness indicator, is proved to be less dissipative than WENO_JS and more computationally efficient than WENO_M. Recently, a less dissipative WENO_Z+ scheme [8] was proposed. The improved scheme attains much better resolution at the smooth region of the solution by adding a new term to the weights of WENO_Z. Ha et al. [9] derived a new method (WENO_NS) that measures the local smoothness of the numerical solution inside a stencil. In this scheme, a parameter is set to compromise between smoothness and discontinuity. Although the numerical results reveal that the new scheme behaves at least the same even better than WENO_JS, WENO_M and WENO_Z scheme, the problem dependent parameter remains hard to be determined for general simulations. In a word, the nonlinear weights of all the above WENO schemes are based on the smoothness measurement. The non-zero smooth indicators will make the stencil adaptation away from the linear optimal stencil and cause numerical dissipation. Taylor et al. [10] developed a hybrid method in which the 5^{th} order WENO_JS and 5^{th} order upwind scheme can be chosen according to the relative smoothness measurement. Numerical results show that the scheme can reduce the dissipation significantly. Later, Li et al. [11] proposed a similar hybrid method (WENO-SYMBO) [12] for direct numerical simulation (DNS) of turbulent boundary layer flow. Hu et al. [13] developed a hybrid WENO_CU to switch between 5^{th} order WENO and 6^{th} order central difference scheme. Similar hybrid scheme (WENO_θ) can be found in [14]. Although the numerical examples suggest that these schemes can preserve shock-capturing properties with very small dissipation, the computational cost of the newly developed smoothness indicators is close to 7^{th} order WENO. Both the WENO_CU and WENO_θ use the 6 points