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A High-Order Modified Finite Volume WENO Method on 3D Cartesian Grids

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Abstract. The modified dimension-by-dimension finite volume (FV) WENO method on Cartesian grids proposed by Buchmüller and Helzel can retain the full order of accuracy of the one-dimensional WENO reconstruction and requires only one flux computation per interface. The high-order accurate conversion between face-averaged values and face-center point values is the main ingredient of this method. In this paper, we derive sixth-order accurate conversion formulas on three-dimensional Cartesian grids. It is shown that the resulting modified FV WENO method is efficient and high-order accurate when applied to smooth nonlinear multidimensional problems, and is robust for calculating non-smooth nonlinear problems with strong shocks.

AMS subject classifications: 65M08, 65M12, 65M20

Key words: Finite volume method, high-order accuracy, dimension-by-dimension reconstruction, Cartesian grid.

1 Introduction

The standard weighted essentially non-oscillatory (WENO) method proposed by Jiang and Shu [1] is widely used for solving hyperbolic conservation laws. The simplest way to use WENO methods on multidimensional Cartesian grids is to apply a one-dimensional WENO scheme in each direction [2]. Conservative finite difference WENO methods based on flux interpolation are used in a dimension-by-dimension fashion and they can retain the full order accuracy of the one-dimensional WENO scheme for linear as well

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as nonlinear multi-dimensional conservation laws. However, in some situations such as adaptively refined Cartesian grids and multi-block Cartesian grids, finite volume methods (FVMs) are more convenient than finite difference methods as FVMs admit a simple formulation around hanging nodes. Unfortunately, while FV WENO methods based on a dimension-by-dimension fashion retain the full order of accuracy for smooth solutions of linear multi-dimensional problems, they are only second-order accurate for smooth solutions of nonlinear multi-dimensional problems [3,4].

A high-order FVM generally includes variable reconstruction within a cell (k-exact reconstruction [5] and its variants [6–11]) and high order flux quadrature on the cell interfaces. On Cartesian grids, the expensive multi-dimensional WENO reconstruction is not necessary. Instead, a series of one-dimensional WENO reconstructions are applied in all directions in order to obtain high-order accurate point values of the conserved quantities at the quadrature points of a cell interface, and then evaluate numerical fluxes at these quadrature points. However, the computational cost of such high-order FV WENO methods on Cartesian grids is still large [3,12,13].

Recently, Buchmüller and Helzel [4] proposed a modification to the dimension-by-dimension FV WENO method on Cartesian grids and applied this modified method on adaptive Cartesian meshes [14,15]. Later on a fourth-order quadrature modification flux (QMF) method was introduced and applied on adaptive Cartesian meshes by Tamaki and Imamura [16]. A key technique used in Refs. [4,14,15] is the conversion between face-averaged values and face-centered values, which helps improve the spatial order of accuracy of the dimension-by-dimension FV WENO method. However, Refs. [4,14] mainly concentrated on two-dimensional problems and Ref. [15] only gave the fourth-order conversion formulas on three-dimensional (3D) Cartesian grids. In this paper, we further develop the modified FV WENO method by deriving sixth-order conversion formulas on 3D Cartesian grids which are not available in Refs. [4,14,15]. The derivation is based on the observation that the differentiation and cell-averaging are exchangeable [16], and it can be extended to even higher order accuracy of conversion. Furthermore, we use the characteristic variables as the reconstructed quantities for the system of conservation laws. For the temporal discretization we use the same Runge-Kutta methods of order fifth or seven as Ref. [4].

The rest of this paper is organized as follows. In Section 2, the modified FV WENO method is explained, and the sixth-order formulas for conversion between face-averaged values and face center point values on 3D Cartesian grids are derived. Numerical results are presented in Section 3 to verify the accuracy, efficiency and robustness of the modified method. Concluding remarks are given in Section 4.

2 Modified finite volume WENO method

In this section we first give the standard dimension-by-dimension FV WENO method, and then derive sixth-order conversion formulas on 3D Cartesian grids. Finally, we build up our modified dimension-by-dimension FV WENO method.