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An Undecomposed Hybrid Algorithm for Nonlinear Coupled Constitutive Relations of Rarefied Gas Dynamics

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Abstract. It is well-established that the accurate simulation of hypersonic rarefied gas flows cannot be accomplished by Navier-Stokes-Fourier (NSF) equations since they are applicable only for small deviations from local thermodynamic equilibrium. To accurately and efficiently solve this problem, a nonlinear-coupled-constitutive-relation (NCCR) model was developed from Eu's generalized hydrodynamic equations, which are consistent with the laws of irreversible thermodynamics. In this paper, Myong's decomposed solver is extended for solving three-dimensional diatomic nonlinear coupled algebraic constitutive equations. Subsequently, a reliable undecomposed hybrid algorithm is proposed for the complete solution of NCCR model through combining the merits of fixed-point and Newton's iterations. The new-developed computational method is validated by high-speed rarefied flows around a cylinder and the Apollo command module. Computation shows that the undecomposed hybrid algorithm makes a great improvement in computational robustness and accuracy. Moreover, this nonlinear constitutive model yields solutions in better agreement with DSMC than NSF model. The results indicate that NCCR model is capable of capturing the physical flow properties away from equilibrium and demonstrate its great potential capability in the further application.

AMS subject classifications: 76K05, 76P05, 82B40

Key words: Generalized hydrodynamics, constitutive relations, modified moment method, decomposed and undecomposed solver, hypersonic rarefied gas flows.

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1 Introduction

Despite recent progress, how to predict aerodynamic characteristics of the hypersonic vehicle accurately and effectively during reentry remains to be elucidated in the field of computational aerodynamics. Multiscale non-equilibrium effects are the main feature during high-speed sustained flight in the rarefied environment, which would bring lots of challenging difficulties to the research. The rarefaction degree can be characterized by the dimensionless Knudsen number, which is defined as the ratio between mean molecule free path and characteristic length. Hypersonic reentry vehicles passing across different flight altitudes would experience several flow regimes: continuum regime, slip regime, transition regime and free molecular flow regime, corresponded to $Kn \leq 0.01$, $0.01 \leq Kn \leq 0.1$, $0.1 \leq Kn \leq 10$, $Kn \geq 10$ respectively. The well-known laws of Navier-Stokes-Fourier (NSF) are applicable only for flows at sufficiently small Knudsen number, in which the mean molecule free path is much smaller than the characteristic lengths of interest. As the flight altitude increases and the relevant length scale can be comparable to the mean free path, NSF equation will lose its superiority, and hence a more refined set of new theoretical analysis tools, beyond the classical theory of linear constitutive relations, need to be developed.

Most work can be categorized into: 'particle simulation theory', 'gas kinetic theory' and 'fluid dynamic models'. They focus on how to get the accurate solution of these challenging flow problems in a physical way. In the former category, one of the most successful and well-known methods has to be the direct simulation Monte Carlo (DSMC) method proposed by G. A. Bird in 1994 [1]. DSMC method depends on tracking statistically representative particles rather than each real gas molecule to simulate the rarefied gas flow. Its great success in predicting the rarefied gas flow, particularly in free molecular flow regime and transition regime, has established the supreme status as an authoritative standard for validating other methods in relative regimes. However, subject to the limited computational resource, its cost is rather prohibitive and unbearable in near continuum regime. To reduce the expensive computational consumption, some hybrid methods, such as NS/DSMC coupling method [2], have been put forward, but how to exchange information and when to switch between continuum and statistical methods still remains to be investigated.

Gas kinetic theory provides an alternative option to describe the non-equilibrium problems. Boltzmann equation, as the central equation in kinetic theory, describes the space-time evolution of a gas distribution function which contains the information of gas atoms at a certain location with microscopic velocities. Boltzmann equation describes the mesoscopic behavior of gases over a spatial scale of molecular mean free path. However, it is a kind of integral-differential equation and the highly nonlinear particle collision term has greatly increased the complexity of the direct solution. Much work in gas kinetic theory is based on the simplification of Boltzmann equation, such as linearization methods of Boltzmann equation [3], modeling equation methods [4]. These reduced kinetic equations can be numerically solved by some well-known conventional methods,