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# FIFA — Fast Interpolation and Filtering Algorithm for Calculating Dyadic Green's Function in the Electromagnetic Scattering of Multi-Layered Structures

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**Abstract.** The dyadic Green's function in multi-layer structures for Maxwell equations is a key component for the integral equation method, but time consuming to calculate. A novel algorithm, the Fast Interpolation and Filtering Algorithm (FIFA), for the calculation of the dyadic Green's function in multi-layer structures is proposed in this paper. We discuss in specific details, ready for use in practical calculations of scattering in layer media, how to apply FIFA to calculate various components of the dyadic Green's function. The algorithm is based on two techniques: interpolation of Green's function both in the spectral domain and spatial domain, and low pass filter window based acceleration. Compared to the popular Complex Image Method (CIM), FIFA provides the same speed and overcomes several difficulties associated with CIM while being more general and robust. Specifically, there are no limitations on the frequency range, the number of layers in the structure and the type of Green's functions to be calculated, and moreover, no need to extract surface wave poles from the spectral form of the Green's function. Numerical results are given to demonstrate the efficiency and robustness of the proposed method.

**Key words**: Fast interpolation and filtering algorithm (FIFA); complex image method (CIM); low pass filter window (LPFW); interpolation table (IT); electromagnetic (EM).

## 1 Introduction

A typical numerical method for electromagnetic (EM) scattering [4, 5, 14] and parameter extraction of VLSI, IC and their packaging [2, 6, 7, 16] is the integral equation (IE) based

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method of moments (MoM) [15]. Discretization using Rao-Wilton-Glission basis functions (RWG) [3] is usually used for the surface IE. The main advantage of the integral formulation is its reduced number of unknowns and flexibility in handling complex geometry of the scatter surface and automatic enforcement of Sommerfeld exterior outgoing conditions via appropriate Green's function [11, 12]. Generally speaking, there are three steps in the computation of such a problem: matrix filling, matrix inversion, and calculation of scattering field and parameter extraction. For a large size problem, the calculation in the second step is dominant and extensive researches have been done in that regard [7, 19–23]. For a moderate size problem, the CPU time used in the first and the third steps, which include the calculation of dyadic Green's function, is much more than that needed for matrix inversions. Therefore, it is necessary to investigate fast evaluation methods for the Green's function. In many cases, such as detection of buried targets, parameter extraction of VLSI and its packaging, numerical modeling of the printed structure used in the monolithic millimeter and microwave circuits (MMIC), the effect of the object's complicated environments can be simply represented by a multi-layered structure model which requires a multi-layered Green's function. The spatial or time domain Green's function for a multi-layered medium can be expressed in the form of a Hankel transform of the spectral Green's function. Though the spectral Green's function can be derived explicitly [8, 11, 12], it is extremely time-consuming to get the time domain Green's function. This is because the kernel of the Hankel transform contains a Bessel function that oscillates fast (especially when the transverse distance between the source point and observation points is large) and decays slowly. There is extensive research literature in the field of fast calculation of the Green's function [9, 10], most of them are based on complex image method (CIM) using the Prony method and the well-known Sommerfeld identity (SI) [17]. CIM is an important useful technique for the Green's function calculation when the source and field points remain in the same layer and the frequency belongs to some limited range. However, in some cases, CIM seems not to work when there is difficulty in extracting the poles from the spectral form of Green's function and when the source and field points are located in different layers. In [4, 5], an efficient evaluation method of the Green's function was proposed, based on the integration along a steepest-descent path (SDP) and leading-order approximations of such integrals. However, it is only valid for a simple structure, e.g. the half space case, in which the SDP can be found. In this paper, we propose and study a robust and general method of fast Green's function calculation for multi-layered structures - the Fast Interpolation and Filtering Algorithm (FIFA). As its name suggests, the method is based on interpolation of the Green's function in the spectral domain (SD) and the time domain (TD), and acceleration by a low- passing filter window function.

The rest of the paper will be organized as follows. In Section 2 we present the specifications of Green's functions for layered structure, an appendix is introduced containing a complete listing of the spectral forms and time domain forms of the components in the dyadic Green's function. A less complete discussion of the spectral Green's function can be found in [11]. To our knowledge, this is the first complete presentation of the potential and electromagnetic field Green's function in both time and spectral domain. Researchers in