COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. 1, No. 2, pp. 261-275

Commun. Comput. Phys.
April 2006

# Solutions of the 1D Coupled Nonlinear Schrödinger Equations by the CIP-BS Method

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Received 30 August 2005; Accepted (in revised version) 25 November 2005

Communicated by Takashi Yabe

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**Abstract.** In this paper, we present solutions for the one-dimensional coupled nonlinear Schrödinger (CNLS) equations by the Constrained Interpolation Profile - Basis Set (CIP-BS) method. This method uses a simple polynomial basis set, by which physical quantities are approximated with their values and derivatives associated with grid points. Nonlinear operations on functions are carried out in the framework of differential algebra. Then, by introducing scalar products and requiring the residue to be orthogonal to the basis, the linear and nonlinear partial differential equations are reduced to ordinary differential equations for values and spatial derivatives. The method gives stable, less diffusive, and accurate results for the CNLS equations.

**Key words**: The CIP-BS method; basis set; differential algebra; the Galerkin method; the coupled nonlinear Schrödinger equation.

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## 1 Introduction

In 1965, the numerical experiment of Zabusky and Kruskal [1] initiated the development of the concept of the soliton and inverse scattering theory. Since then, many numerical

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methods have been proposed to elucidate complicated processes by accurately solving the nonlinear partial differential equations (PDEs). The methods belong essentially to one of two classes: spectral methods and grid methods. The main difference between these two methods comes from the methodology in treating the spatial derivatives. With a spectral method the solution is approximated by some finite linear combination of differentiable basis functions, each one of which satisfies the boundary conditions. The derivatives in a spectral method do not suffer from numerical inaccuracies. With a grid method, such as the finite element method or the finite difference method, the derivatives are approximated by some differences. It is often difficult to approximate the derivatives with sufficient accuracy, because the derivatives are estimated by using only the values of the function on a compact set of grid points. In general, spectral methods give accurate solutions with a minimum number of discretization points, only if appropriate problem specific basis functions are applicable. However, a finite difference method is typically more flexible and easier to implement than a spectral method for systems with complex boundary conditions. By considering the merits and demerits of each method, it is believed that the method in which the solution is expanded by a finite number of local differentiable basis functions is to be preferred.

As far as incorporating only the values at grid points, it seems difficult to improve grid methods which exemplify the spectral method's accuracy. Recently, a new numerical method, the CIP-Basis Set (CIP-BS) method [2–4], has been proposed by generalizing the concept of the Constrained Interpolation Profile (CIP) method [5, 6] from the viewpoint of the basis set. The idea of the CIP method is that not only values but also their first derivatives are treated as independent variables associated with the grid points, and the information lost inside the grid cell is retrieved by a Hermite type interpolation function [7]. The CIP method has been successfully applied to various complex linear and nonlinear hydrodynamic problems, covering both compressible and incompressible flow [8], such as shock wave generation, laser-induced evaporation, and elastic-plastic flow. However, the methods using matrix operations are advantageous in investigating the characteristics of the system, and a number of numerical methods for large, sparse systems developed for the finite difference method or the finite element method can be adopted. With this view, the CIP-BS method has introduced a polynomial basis set, by which physical quantities are approximated with their values and derivatives associated with grid points. The governing equations are discretized into matrix form equations requiring the residuals to be orthogonal to the basis functions via the same principle as the Galerkin method. The CIP-BS method, in which the local polynomial basis functions corresponding to the values and spatial derivatives at each grid point belong to the complete set and the class $\mathrm{C}^K$, is called the CIP-BS$^K$ method. Numerical results in the solution of the linear Schrödinger equation have demonstrated that accurate solutions are obtained by the method and that the use of a higher order basis set is essential in increasing accuracy.

The purpose of this paper is to show that the CIP-BS method can be extended for nonlinear operations on functions in the framework of differential algebra, and can be a universal solver of nonlinear PDEs by exemplifying the solutions of the one-dimensional