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On the Reduction of Numerical Dissipation in Central-Upwind Schemes

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Abstract. We study central-upwind schemes for systems of hyperbolic conservation laws, recently introduced in [13]. Similarly to staggered non-oscillatory central schemes, these schemes are central Godunov-type projection-evolution methods that enjoy the advantages of high resolution, simplicity, universality and robustness. At the same time, the central-upwind framework allows one to decrease a relatively large amount of numerical dissipation present at the staggered central schemes. In this paper, we present a modification of the one-dimensional fully- and semi-discrete central-upwind schemes, in which the numerical dissipation is reduced even further. The goal is achieved by a more accurate projection of the evolved quantities onto the original grid. In the semi-discrete case, the reduction of dissipation procedure leads to a new, less dissipative numerical flux. We also extend the new semi-discrete scheme to the two-dimensional case via the rigorous, genuinely multidimensional derivation. The new semi-discrete schemes are tested on a number of numerical examples, where one can observe an improved resolution, especially of the contact waves.

AMS subject classfications: 65M10, 65M12, 35L65

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1 Introduction

Consider the systems of hyperbolic conservation laws:

$$\frac{\partial}{\partial t}\mathbf{u}(\mathbf{x},t)+\nabla_{\mathbf{x}}\cdot\mathbf{f}(\mathbf{u}(\mathbf{x},t))=\mathbf{0}, \tag{1.1}$$

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where $\mathbf{u}(\mathbf{x},t)=(u_1(\mathbf{x},t),\ldots,u_N(\mathbf{x},t))^T$ is an N-dimensional vector and $\mathbf{f}$ is a nonlinear convection flux. In the general multidimensional case, $\mathbf{u}$ is a vector function of a time variable t and d-spatial variables $\mathbf{x}=(x_1,\ldots,x_d)$ with the corresponding fluxes $\mathbf{f}=(f^1,\ldots,f^d)$. Such systems arise in many different applications, for instance, in fluid mechanics, geophysics, meteorology, astrophysics, financial and biological modeling, multi-component flows, groundwater flow, semiconductors, reactive flows, geometric optics, traffic flow, and other areas.

We study numerical methods for system (1.1). In particular, we are interested in Godunov-type finite-volume central schemes, which are *simple, robust* and *universal* Riemann-problem-solver-free methods for general systems of conservation laws. The key idea in their construction is the integration over the control volumes that contain the entire Riemann fans. Such a setting allows one to evolve a piecewise polynomial projections of the computed solution to the next time level without (approximately) solving (generalized) Riemann problems, arising at cell interfaces.

The prototype central scheme is the celebrated (staggered) Lax-Friedrichs scheme [3, 19]. This is a very dissipative first-order scheme, which typically cannot provide a satisfactory resolution of discontinuities and rarefaction corners unless a very large number of grid points is used. Its higher-order (and also higher resolution) generalization was proposed by Nessyahu and Tadmor in [28]. Later the one-dimensional (1-D) second-order Nessyahu-Tadmor scheme was extended to higher orders [22,26] and to more than one spatial dimensions [1,7,24]. Its nonstaggered version was proposed in [6].

The major drawback of staggered central schemes is their relatively large numerical dissipation. This makes them inappropriate for large time integrations, steady-state computations, and for the cases where small time steps are enforced, for example, due to the presence of source or diffusion terms on the right-hand side (RHS) of (1.1). They also do not admit a semi-discrete form, which may be particularly advantageous in the latter case (see, e.g., [8,10,12,17]).

In order to eliminate the aforementioned disadvantages, a new class of high-resolution central schemes has been recently proposed in [17]. The main idea in the construction of the new central schemes is to utilize the *local propagation speeds* to obtain a more precise estimate on the width of Riemann fans. The solution is then evolved separately in "nonsmooth" (those that include Riemann fans) and "smooth" control volumes, and the resulting nonuniformly distributed data are projected back onto the original, *non-staggered* grid. The higher-order extensions of this scheme were proposed in [11, 14, 18], and its genuinely multidimensional generalization was obtained in [14].

The numerical dissipation present at central schemes can be further reduced by utilizing *one-sided* local propagation speeds. This leads to the so-called *central-upwind schemes*, recently introduced in [13]. This class of schemes enjoys all the major advantages of Riemann-problem-solver-free central schemes — high efficiency, simplicity and universality. At the same time, it has a certain upwind nature (more information on the directions of wave propagation is utilized since the control volumes over the Riemann fans are no longer symmetric), which leads to a higher resolution.