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Simulation of Earthquake Rupture Dynamics in Complex Geometries Using Coupled Finite Difference and Finite Volume Methods

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Abstract. We couple a node-centered finite volume method to a high order finite difference method to simulate dynamic earthquake ruptures along nonplanar faults in two dimensions. The finite volume method is implemented on an unstructured mesh, providing the ability to handle complex geometries. The geometric complexities are limited to a small portion of the overall domain and elsewhere the high order finite difference method is used, enhancing efficiency. Both the finite volume and finite difference methods are in summation-by-parts form. Interface conditions coupling the numerical solution across physical interfaces like faults, and computational ones between structured and unstructured meshes, are enforced weakly using the simultaneous-approximation-term technique. The fault interface condition, or friction law, provides a nonlinear relation between fields on the two sides of the fault, and allows for the particle velocity field to be discontinuous across it. Stability is proved by deriving energy estimates; stability, accuracy, and efficiency of the hybrid method are confirmed with several computational experiments. The capabilities of the method are demonstrated by simulating an earthquake rupture propagating along the margins of a volcanic plug.

AMS subject classifications: 35L05, 35L65, 35Q35, 65M06, 65M08, 65M12, 65Z05

Key words: Elastic waves, earthquake, high order finite difference finite volume, summation-by-parts, simultaneous approximation term, nonlinear boundary conditions.

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1 Introduction

Computational modeling of earthquake rupture dynamics presents many challenges. Like similar radiation problems in electrodynamics and other fields, there is particular interest in waves in the far field, as most observations are made at distances many wavelengths away from a compact source region. This argues for the use of high order methods with minimal dispersion errors. However, in dynamic rupture models, the source process itself is not known a priori, but is determined as part of the solution. To be more specific, seismic (i.e., elastic) waves are generated by slip across fault surfaces (i.e., the discontinuity in the tangential component of the displacement field across an internal interface). Slip on one part of the fault excites waves that transmit stresses to adjacent parts of the fault, possibly triggering slip there and leading to the progressive propagation of a rupture. The condition for fault slip is typically expressed as a nonlinear friction law coupling fault slip velocity and tractions acting on the sides of the fault. Further challenges arise from the geometrical complexity of natural fault geometries, often involving multiple nonplanar surfaces with kinks and branches. Numerical methods based on unstructured meshes are well suited to handle this level of complexity in the near field source region. The challenge, then, is to combine the advantages of numerical methods based on unstructured meshes (for the near field or source region) with high order numerical methods based on structured grids (for the far field region), in an accurate and stable manner.

A variety of other numerical approaches have been taken to study earthquake rupture dynamics, each with benefits and shortcomings. Some of the more recent numerical approaches we will describe have been, or are actively being, verified and evaluated using a series of benchmark exercises as part of the Southern California Earthquake Center/U.S. Geological Survey (SCEC/USGS) Dynamic Earthquake Rupture Code Verification Project [28]. Traditionally, finite difference methods have been widely used, but mostly for planar faults (e.g., [3, 16, 42, 44, 69]). In more recent years, nonplanar fault geometries have also been incorporated in finite difference methods using coordinate transform techniques (e.g., [14, 15, 36]). In the coordinate transform method developed in [36], the physical domain is decomposed into multiple curvilinear blocks that conform to nonplanar surfaces. Each block is mapped onto a rectangle or square in the computational domain, and the transformed equations are solved in the computational domain with finite differences. Severe grid skewness, for instance due to intersecting faults with small angles, can cause the transform to be poorly conditioned. Thus, it can be difficult to develop well-conditioned multi-block decompositions of geometries that arise in realistic fault systems. Boundary element methods have also been developed (e.g., [4, 22, 32, 64, 70]). Solutions given by these methods are limited to faults in a uniform medium and some can develop numerical instabilities. These methods can handle nonplanar faults, except for the spectral boundary integral equation method [22]. However, the spectral boundary integral equation method is quite efficient and accurate for planar fault problems, and has been widely used to investigate realistic fault weakening