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Three-Point Combined Compact Alternating Direction Implicit Difference Schemes for Two-Dimensional Time-Fractional Advection-Diffusion Equations

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Abstract. This paper is devoted to the discussion of numerical methods for solving two-dimensional time-fractional advection-diffusion equations. Two different three-point combined compact alternating direction implicit (CC-ADI) schemes are proposed and then, the original schemes for solving the two-dimensional problems are divided into two separate one-dimensional cases. Local truncation errors are analyzed and the unconditional stabilities of the obtained schemes are investigated by Fourier analysis method. Numerical experiments show the effectiveness and the spatial higher-order accuracy of the proposed methods.

AMS subject classifications: 65M06, 65M12, 65N06, 65N12

Key words: Time-fractional advection-diffusion equations, combined compact difference (CCD) scheme, ADI, stability, Fourier analysis.

1 Introduction

Fractional calculus has been widely studied recently since its numerous practical applications have not been realized until a couple of last decades [1]. It has found increasing place in wide field of applications including control system community, turbulent flows, viscoelasticity, plasma physics, thermodynamics, anomalous diffusion in porous media, finance theory and so on. These applications prompted the emergence of various fractional differential equations in mathematical and physical world [1–3]. To seek the solutions of these equations becomes an essential part of mathematicians' work. The

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integral transform method, power series method and Green's function method are employed to produce the analytical solution of some fractional differential equations. Consequently, the solution usually refers to some special functions which are quite complicated in real calculation. Numerical solutions of fractional differential equations are preferred by many scholars.

The time-fractional derivative is used to describe the stochastic process with non-Markovian property, or the anomalous diffusion process with memory. The differential equation with presence of the time-fractional derivative is called the time-fractional differential equation. The huge amount of works focused on how this class of equation is numerically solved, among which the finite difference method was extensively applied. For one-dimensional time-fractional differential equations, Yuste and Acedo [4], Langlands and Henry [5], Chen et al. [6], Sun and Wu [7] developed various finite difference methods with the second-order accuracy in space. To promote the spatial accuracy, some fourth-order compact finite difference methods were proposed successively. Cui [8], Gao and Sun [9], Zhang, Sun and Wu [10], Mohebbi and Abbaszadeh [11] focused on the study of spatial fourth-order accurate finite difference methods for solving the time-fractional subdiffusion equations. More relevant works cover the literatures by Du, Cao and Sun [12] for the second-order fractional wave equation, Hu and Zhang [13] for the fourth-order diffusion-wave system, Ren and Sun [14] for the second-order fractional wave equation with Neumann boundary conditions. In addition, the finite difference discretization in time and spectral approximation in space was reported in [15] for time-fractional diffusion equations.

As we know, the time-fractional derivative is a nonlocal operator of convolution type. Values on current time level depend on values on all previous time levels. The variable storage and computational cost appear an increasing burden as time grows, especially during simulating long time behaviors of solutions. Hence, it is worth developing some higher-order accurate numerical methods for solving the time-fractional differential equations. For high dimensional problems, this requirement is more outstanding. Some relevant works have been done along this way. For two-dimensional anomalous subdiffusion equations, Brunner, Ling and Yamamoto [16] discussed an algorithm by coupling an adaptive time stepping and adaptive spatial basis selection approach. Chen et al. [17] proposed two numerical methods for this problem: one was implicit and the other was explicit, based on Grünwald-Letnikov approximation for the time-fractional derivative. Fourier analysis method was used to discuss their stability and convergence. Recently, the ADI technique was introduced to handle the two-dimensional time-fractional differential equations. Zhang and Sun [18] presented two different ADI schemes for the 2D time-fractional sub-diffusion equation. The stability and convergence of the resulting schemes were analyzed by the discrete energy method. The obtained schemes in [17, 18] were both second-order accurate in space. To improve the numerical accuracy, Cui [19,20] investigated high-order compact ADI schemes for the 2D time-fractional sub-diffusion equation. Zhang, Sun and Zhao [21], Zhang and Sun [22] constructed compact difference schemes for the 2D time-fractional wave equation and sub-diffusion equation, respec-