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The Diffused Vortex Hydrodynamics Method

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Abstract. A new Particle Vortex Method, called Diffused Vortex Hydrodynamics (DVH), is presented in this paper. The DVH is a meshless method characterized by the use of a regular distribution of points close to a solid surface to perform the vorticity diffusion process in the boundary layer regions. This redistribution avoids excessive clustering or rarefaction of the vortex particles providing robustness and high accuracy to the method. The generation of the regular distribution of points is performed through a packing algorithm which is embedded in the solver. The packing algorithm collocates points regularly around body of arbitrary shape allowing an exact enforcement on the solid surfaces of the no-slip boundary condition. The present method is tested and validated on different problems of increasing complexities up to flows with Reynolds number equal to 100,000 (without using any subgrid-scale turbulence model).

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Key words: Vorticity dynamics, vortex method, particle methods, meshless methods, meshfree methods, packing algorithm, diffused vortex hydrodynamics, body fitted method, vortex wake.

1 Introduction

This work deals with the modelling of viscous incompressible flows using a new particle vortex method called Diffused Vortex Hydrodynamics (DVH).

The use of a vortex method brings several advantages (see *e.g.* [4,9,18]). Firstly, using the vorticity formulation of the Navier-Stokes equation the pressure field is no longer a direct unknown of the problem. The vorticity formulation also allows to discretize with

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vortex particles only the rotational region of the flow. Secondly, the boundary conditions at infinity are automatically satisfied, therefore large spatial domains are not required to correctly enforce these boundary conditions. Another advantage is the possibility to move the particles in a Lagrangian way, reducing the numerical dissipation always present with mesh based approach. On the other hand, the pure Lagrangian nature of this method brings also a drawback: a "redistribution techniques" must be introduced in order to preserve the accuracy of the method.

The vortex methods are classified mainly by:

(i) the nature of the discretization used for the vortical region,

(ii) the methods used for the evaluation of the velocity field from the vorticity one,

(iii) the computation of the viscous diffusion process.

The DVH solver presented here is designed to be meshless. As an example of the existing techniques consider the Vortex-in-Cell (ViC) method. In ViC method the velocity field can be solved using highly efficient hybrid particle-mesh algorithms and taking advantage of fast solvers and multigrid methods (see [6]).

Conversely, in DVH the velocity field is obtained using a fast multipole method (FMM), allowing for a complete mesh-free approach for the advection process. The diffusion process does not require a mesh either. A single time step is subdivided into advective and diffusive sub-steps thanks to a splitting algorithm [4]. The diffusion is performed following the deterministic algorithm by Benson et al. [3], by a superposition of elementary solutions of the heat equation. In the present scheme, during diffusion, each vortex particle gives a vorticity contribution on a "Regular Point Distribution" (RPD). This special set of points will then become the new set of vortex particles overwriting the previous one. This procedure prevents the excessive clustering or rarefaction of the vortex particles, and avoids the remeshing procedure which would be required otherwise (see *e.g.* [2]). This problem is common also in the context of other particle methods, for example in the Particle-in-Cell method (the reader can refer to the recent work of *E. Edwards & R. Bridson (2012)* [11]), where the accuracy of the simulations is linked to the interpolation procedure from the particles set to the mesh used for solving the governing equations.

Because there is no any topological connection between the points of the RPDs, the DVH scheme can be considered a pure meshless method.

In the region close to solid bodies, the RPDs are generated using the particle packing algorithm described in [7]. This algorithm is very fast and efficient and, since it is based on particle-dynamic interactions, it can be directly embedded in the same code.

For the regions far from the solid surfaces simple Cartesian lattices, with different sizes according to the distance from the body, can be used as RPDs. In the DVH model, all the different RPDs can overlap each other and can be characterized by different spatial resolutions. Each vortex particle is associated to a RPD at each time step, and during the diffusion each vortex spreads its circulation on the associated RPD.