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On Initial Conditions for the Lattice Boltzmann Method

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Abstract. In this paper, we propose two initialization techniques for the lattice Boltzmann method. The first one is based on the theory of asymptotic analysis developed in [M. Junk and W.-A. Yong, Asymptotic Anal., 35(2003)]. By selecting consistent macroscopic quantities, this initialization leads to the second-order convergence for both velocity and pressure. Another one is an improvement of the consistent initial conditions proposed in [R. W. Mei, L.-S. Luo, P. Lallemand and D. d'Humières, Comput. Fluids, 35(2006)]. The improvement involves a modification of the collision term and a reconstruction step. Numerical examples confirm the accuracy and efficiency of our techniques.

AMS subject classifications: 65M06, 76M20

Key words: Lattice Boltzmann method, initial conditions, asymptotic analysis, incompressible Navier-Stokes equation.

1 Introduction

In the past two decades, the lattice Boltzmann method (LBM) [3, 4, 7, 11–13, 19, 26, 27] has emerged as a powerful tool for simulating various fluid flow problems. Because of its simplicity, effectiveness and massive parallelism, the method has been widely used in many flow simulations including thermal flows [9, 10, 21, 25], multi-phase and multi-component flows [22, 28], porous media flows [5, 24], *etc*.

In the meantime, the theoretical study of the LBM has been greatly developed. In the early literature, the macroscopic fluid dynamic equations were obtained from the LBM by using the multi-scale technique [19, 26]. Later, the LBM was regarded as a special discretization of the continuous Boltzmann equation [12, 13]. Furthermore, some important properties (*e.g.* stability conditions, dispersion effects) of the LBM were studied [20].

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After that, the non-existence of H theorems was proved for some lattice Boltzmann models [32,33]. Recently, a theory of asymptotic analysis, including stability and convergence, for the method were established in a mathematically rigorous fashion [1,15–18,30,34].

Initial conditions are indispensable parts of the LBM. For practical applications, available initial data are only those of macroscopic quantities, *e.g.* density, velocity and temperature, instead of the distribution functions $f_i(t,x)$ in the LBM. A conventional way is to specify the initial distributions $f_i(0,x)$ as equilibrium ones $f_i^{eq}(\rho_0,u_0)$ with (ρ_0,u_0) the initial data of density and velocity. But this seems not correct for flows with strong nonlinearity [35] and can lead to initial layers and thereby poor numerical resolutions. Because of this, a non-equilibrium correction method based on the Chapman-Enskog expansion was proposed [6,29] to improve the initialization. As pointed out in [23], such methods are cumbersome and not easy to implement in practice since it involves high-order time and spatial derivatives. Another drawback of the previous methods is that the initial density field ρ_0 is required but often not available. To overcome this difficulty, an iterative initialization procedure was proposed in [2,23] to generate consistent initial conditions. This idea is very effective. However, the pressure is only of first-order accuracy, which seems to be the maximal accuracy for problems with general non-periodic boundary conditions in the literature. We also remark that Van Leemput et al. developed an alternative approach to generate initial conditions for LBM [31].

In this work, we propose two kinds of initializations for the LBM. The first one is obtained by using the theory of asymptotic analysis developed in [16–18] for the LBM. Since high-order corrections are involved, the initial layers can be avoided. Here no time derivatives is involved, quite different from the non-equilibrium correction method [29], so it is easy for implementation. Moreover, the velocity and pressure converge at second order since the macroscopic quantities are chosen in a consistent way. The other one is an improvement of the consistent initial conditions proposed in [23]. It significantly decrease the iteration steps and numerical errors by modifying the collision term and adding an extra reconstruction step. Numerical results show that the new initialization routines significantly improve the existing methods.

The paper is organized as follows. In Section 2, after a brief review on the LBM and its asymptotic analysis theory, we present the high order initial conditions for the LBM. Motivated by the asymptotic analysis, we propose an improvement of the consistent initial conditions in Section 3. In Section 4, numerical examples are provided to show the performance of our methods. Some conclusive remarks are made in Section 5.

2 LBM and its asymptotic analysis

2.1 The lattice Boltzmann method

The general form of the LBM is

$$f_i(t+h^2,x+c_ih)=f_i(t,x)+J_i(\vec{f}(t,x))+\widehat{F}_i, \qquad i=1,2,\cdots,N. \tag{2.1}$$