

# Immersed Boundary – Thermal Lattice Boltzmann Methods for Non-Newtonian Flows Over a Heated Cylinder: A Comparative Study

A. Amiri Delouei<sup>1</sup>, M. Nazari<sup>1,\*</sup>, M. H. Kayhani<sup>1</sup> and S. Succi<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, University of Shahrood, Shahrood, Iran.

<sup>2</sup> IAC-CNR, Rome, Via dei Taurini 19, 00185, Roma & Department of Physics, Harvard University, Oxford Street 60, Cambridge, MA 02138, USA.

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**Abstract.** In this study, we compare different diffuse and sharp interface schemes of direct-forcing immersed boundary — thermal lattice Boltzmann method (IB-TLBM) for non-Newtonian flow over a heated circular cylinder. Both effects of the discrete lattice and the body force on the momentum and energy equations are considered, by applying the split-forcing Lattice Boltzmann equations. A new technique based on predetermined parameters of direct forcing IB-TLBM is presented for computing the Nusselt number. The study covers both steady and unsteady regimes ( $20 < Re < 80$ ) in the power-law index range of  $0.6 < n < 1.4$ , encompassing both shear-thinning and shear-thickening non-Newtonian fluids. The numerical scheme, hydrodynamic approach and thermal parameters of different interface schemes are compared in both steady and unsteady cases. It is found that the sharp interface scheme is a suitable and possibly competitive method for thermal-IBM in terms of accuracy and computational cost.

**AMS subject classifications:** 65Y20, 76A05, 76D25, 76P05, 76R05

**Key words:** Immersed Boundary Method, Thermal Lattice Boltzmann Method, non-Newtonian fluid, cylinder, power-law fluids.

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## 1 Introduction

The problem of Newtonian fluid flow and heat transfer over a cylinder has been studied by several researchers as a benchmark for verification of solution methods in complicated geometries [1–3]. This flow configuration provides valuable insights into the

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\*Corresponding author. Email addresses: a.a.delouei@gmail.com (A. A. Delouei), mnazari@shahroodut.ac.ir (M. Nazari), h\_kayhani@shahroodut.ac.ir (M. H. Kayhani), s.succi@iac.cnr.it (S. Succi)

nature of the fundamental physical interaction phenomena involving momentum and heat transfer, [4]. Due to the abundance of industrial applications, such as the formation of weld lines in polymer processing operations [5], the use of thin cylinders and wires as measurement probes and sensors in non-Newtonian flows [6], in the resin transfer molding process of manufacturing fiber reinforced composites [6], thermal treatment of various food-stuffs [4], fluidized bed drying of fibrous substances [4], non-Newtonian fluids flows have entered into this discussion. In comparison with the work developed in the field of non-Newtonian fluids flow in presence of immersed body [7], the thermal investigation of these fluids is less extended, even for the flow of Newtonian fluids over a heated circular cylinder [8]. In fact, the few studies in this area have employed finite volume methods using FLUENT software [8–10] or finite difference methods [11].

Recently, some progress, based on the immersed boundary idea of Peskin [12], has been accomplished, which can simplify the solution in complex geometries. The immersed boundary method (IBM) was first proposed by Peskin [12] to investigate flow patterns around heart valves, without any body-fitted mesh. Over the years, the IBMs have been improved and overcome most of the original limitations of original Peskin method [13, 14]. At present, the IBM has gained a prominent role for the numerical simulation of complex fluid-structure interactions [15]. The IBM can be categorized both as a mathematical formulation and as a numerical scheme. A combination of Eulerian and Lagrangian variables are considered in the mathematical formulation. An interaction equation linked these two types of variables. In the numerical aspect of IBM, the Eulerian variables are denoted by fixed Cartesian mesh while, the Lagrangian variables are traced via a completely separated curvilinear mesh, with the ability of moving freely across the fixed Eulerian nodes (see Fig. 1). Generally speaking, the IBM is a non-body-conformal grid method that satisfies the no-slip boundary condition (or thermal boundary condition) by implementing a force density (or an energy-force density) term within the flow governing equation (or the energy equation). With respect to the algorithm of the boundary force evaluation, at least three general groups of IB methods can be identified: (i) feedback-forcing method, (ii) direct-forcing method, and (iii) momentum exchange-based method. The former employs the feed-back process based on the position (and/or velocity, temperature) of the boundary points [12] and the second one utilizes the flow equation (or energy equation) to regulate the boundary force (or boundary energy-force) density [17]. In the third group (for example [18, 19]), the forcing term is computed via the momentum exchange method introduced by Ladd [20].

The dependency of two non-coincident Eulerian and Lagrangian grids can be established by interface schemes based on a smoothed approximation of the Dirac delta function (diffuse interface scheme) [21] or a sharp interpolations (sharp interface schemes) [22]. The choice of the appropriate interface scheme is another very important issue beside the force evaluation method in the IBM [23]. We shall present detailed comparisons in the following.

As mentioned above, in the IBM, the Eulerian grids should be placed in a Cartesian frame; the Lattice Boltzmann Method (LBM) is an excellent solver which satisfies this