Commun. Comput. Phys.
doi: 10.4208/cicp.131014.030615a

Vol. **19**, No. 2, pp. 301-328
February 2016

Asymptotic-Preserving Scheme for the M_1-Maxwell System in the Quasi-Neutral Regime

S. Guisset[1,2,*], S. Brull[1], B. Dubroca[2], E. d'Humières[2], S. Karpov[3] and I. Potapenko[3]

[1] *Univ. Bordeaux, IMB, UMR 5251, F-33405 Talence, France.*
[2] *Univ. Bordeaux, CELIA, UMR 5107, F- 33400 Talence, France.*
[3] *Keldysh Institute for Applied Mathematics, 125047 Moscow, Russian Federation.*

Received 13 October 2014; Accepted (in revised version) 3 June 2015

Abstract. This work deals with the numerical resolution of the M_1-Maxwell system in the quasi-neutral regime. In this regime the stiffness of the stability constraints of classical schemes causes huge calculation times. That is why we introduce a new stable numerical scheme consistent with the transitional and limit models. Such schemes are called Asymptotic-Preserving (AP) schemes in literature. This new scheme is able to handle the quasi-neutrality limit regime without any restrictions on time and space steps. This approach can be easily applied to angular moment models by using a moments extraction. Finally, two physically relevant numerical test cases are presented for the Asymptotic-Preserving scheme in different regimes. The first one corresponds to a regime where electromagnetic effects are predominant. The second one on the contrary shows the efficiency of the Asymptotic-Preserving scheme in the quasi-neutral regime. In the latter case the illustrative simulations are compared with kinetic and hydrodynamic numerical results.

AMS subject classifications: 82D10, 82B40, 82C40

Key words: Asymptotic-Preserving scheme, Fokker-Planck-Landau equation, Maxwell equations, quasi-neutral limit, angular M_1 moments model.

1 Introduction

This work deals with non-homogeneous collisional plasmas described by a kinetic model. The plasma is considered as a mixture of electrons and ions. Each species is characterised by its distribution function which corresponds to particles density in the phase space. In this work the M_1 angular moments model describes electron transport and considers binary collisions between particles whereas Maxwell equations are used to describe the

*Corresponding author. *Email address:* `guisset@celia.u-bordeaux1.fr` (S. Guisset)

evolution of electromagnetic fields. For the sake of simplicity, we assume that the plasma consists of electrons and one ion species considered as immobile. This approximation is relevant due to the important mass of ions compared to the electrons mass. This means our model is valid on time scales during which the ions motion can be neglected.

For the study of collisional processes, the two important physical scales are the mean free path and the electron-ion collision frequency. The mean free path represents the average distance travelled by an electron between two collisions with an ion. The electron-ion collision frequency represents the number of electron-ion collision per unit of time. When the electronic plasma period is very small compared to the electron-ion collisional period and the Debye length is very small compared to the mean free path, the plasma is designated as quasi-neutral and the Maxwell-Gauss (also called Maxwell-Poisson) and Maxwell-Ampere equations degenerate into algebraic equations on collisional time scales.

Therefore to handle this type of situation a new class of methods, called Asymptotic-Preserving (AP) methods has been developed. These methods have been introduced firstly by Shi Jin [25] in the context of diffusive limits for kinetic equations. Consider a system (S_α) depending on a parameter α and (S_0) the corresponding limit system when α tends to zero. In our case α is the ratio between the Debye length and the mean free path. A numerical scheme with time step Δt and space step Δx is called Asymptotic-Preserving in the limit α tends to zero for the system (S_α) if the scheme is stable independently of the values taken by α and if the limit scheme obtained for $\alpha = 0$ is consistent with the limit problem (S_0). In this work the system (S_α) corresponds to the Fokker-Planck-Landau-Maxwell system and (S_0) corresponds to the Fokker-Planck-Landau-Maxwell system in the quasi-neutral limit. This regime has been already studied in the context of fluid models [10,12,17]. For example in [12], the authors considered a two fluid isentropic Euler system coupled with the Poisson equation. It is shown that the Maxwell-Poisson equation can be reformulated into an elliptic equation which does not degenerate at the quasi-neutral limit. In [11], this approach is generalised to the Euler-Maxwell model with a strong magnetic field. A kinetic model consisting in a two fluid Vlasov-Poisson system has also been investigated in [14]. In [16], an Asymptotic-Preserving scheme is proposed for the Euler-Maxwell system in the quasi-neutral regime. The Maxwell equations are reformulated to enable the computation of the electrostatic field even in the limit regime. The development followed to express the electric field is well known in physics [3, 9].

The present paper deals with the construction of an Asymptotic-Preserving scheme or the M_1-Maxwell system in the quasi-neutral limit. The strategy adopted is similar to the one in [16], nevertheless to our knowledge, it is the first time that such schemes are considered for kinetic models with true collision operators. This fact is very important to deal with collisional plasma because the collision frequency ν must follow the Coulombian interaction law $(\nu \approx 1/|v|^3)$. To perform realistic simulations in plasma physics, Coulombian interactions must be used. Therefore, relaxation operators are not relevant from a physical point of view. Moreover up to now, Asymptotic-Preserving schemes for the quasi-neutral limit have been developed either for fluid description of plasma or for collisionless plasmas. Asymptotic-Preserving schemes have been recently used for nu-