Commun. Comput. Phys.
doi: 10.4208/cicp.211015.040316a

Vol. **20**, No. 2, pp. 301-324
August 2016

Finite Volume Lattice Boltzmann Method for Nearly Incompressible Flows on Arbitrary Unstructured Meshes

Weidong Li[1,2] and Li-Shi Luo[2,1,*]

[1] *Department of Mathematics & Statistics, Old Dominion University, Norfolk, Virginia 23529, USA.*
[2] *Computational Science Research Center, Beijing 100193, China.*

Received 21 October 2015; Accepted (in revised version) 4 March 2016

Abstract. A genuine finite volume method based on the lattice Boltzmann equation (LBE) for nearly incompressible flows is developed. The proposed finite volume lattice Boltzmann method (FV-LBM) is grid-transparent, *i.e.*, it requires no knowledge of cell topology, thus it can be implemented on arbitrary unstructured meshes for effective and efficient treatment of complex geometries. Due to the linear advection term in the LBE, it is easy to construct multi-dimensional schemes. In addition, inviscid and viscous fluxes are computed in one step in the LBE, as opposed to in two separate steps for the traditional finite-volume discretization of the Navier-Stokes equations. Because of its conservation constraints, the collision term of the kinetic equation can be treated implicitly without linearization or any other approximation, thus the computational efficiency is enhanced. The collision with multiple-relaxation-time (MRT) model is used in the LBE. The developed FV-LBM is of second-order convergence. The proposed FV-LBM is validated with three test cases in two-dimensions: (a) the Poiseuille flow driven by a constant body force; (b) the Blasius boundary layer; and (c) the steady flow past a cylinder at the Reynolds numbers Re = 10, 20, and 40. The results verify the designed accuracy and efficacy of the proposed FV-LBM.

AMS subject classifications: 76M12, 76M28, 76D05, 82C40

PACS: 47.11.Df, 47.11.Qr, 47.10.ad, 51.10.+y

Key words: Finite volume method, lattice Boltzmann equation, arbitrary unstructured mesh, complex geometry, multi-dimensional scheme.

1 Introduction

In its simplest form, the orthodox lattice Boltzmann method (LBM) is associated with uniform Cartesian meshes due to its tightly coupled discretizations of phase space and

*Corresponding author. *Email addresses:* `lwd_1982.4.8@163.com` (W. D. Li), `lluo@odu.edu`, `lluo@csrc.ac.cn` (L.-S. Luo)

time [1,2]. In spite of its accuracy and algorithmic simplicity [3–7], the capability for the LBM to *accurately* treat flows with complicated geometries is limited. To overcome this limitation, there have been continuous attempts to develop the lattice Boltzmann equation (LBE) based on finite volume (FV) formulation with unstructured meshes [8–17]. The finite-volume method (FVM) for solving the Navier-Stokes equations is a matured technique (cf., *e.g.*, the review [18] and monograph [19]). A key feature of the FVM is its use of *unstructured* meshes to treat complex geometries with ease [18]. With edge-based (2D) or face-based (3D) data structure, cell-centered FVM can be efficiently implemented. The characteristics of the FVM is much determined by its reconstruction step, in which the fluxes are reconstructed at the cell boundaries from the hydrodynamic variables at cell centers, and the hydrodynamic variables are also allowed to be discontinuous at cell boundaries. This reconstruction step distinguishes FVM from other methods, such as finite difference method (FDM) and finite element method (FEM) and so on.

To justify the present work, we begin with a brief survey of existing work on the development of finite-volume lattice Boltzmann method (FV-LBM) for nearly incompressible flows with low Mach numbers. In the work by Nannelli and Succi [8], the fluxes of the distribution functions at cell boundaries are *directly* computed from their values at cell centers through interpolations without the aforementioned reconstruction step. Though this scheme increases geometrical flexibility, it is only implemented on meshes of quadrilateral cells. To remove this limitation, Peng *et al.* [9,10] extend the scheme to a vertex-centered FV-LBM on triangular meshes. However, none of these schemes [8–10] and their variants [11–13,15,17] is *bona fide* FVM for they lack the centerpiece of the FVM — the reconstruction of fluxes at cell boundaries; they are essentially some recasts of some finite difference schemes [13]. These schemes are severely limited in time step size, thus they are computationally inefficient [13].

More recently, Patil and Lakshmisha developed a genuine FV-LBM on triangular unstructured meshes [16]. The reconstruction in this work uses the least-square method to obtain the gradients at cell centroids and Roe's flux-difference splitting scheme to compute the fluxes at cell boundaries. In addition, the techniques of total variation diminishing (TVD) and limiters are used in the scheme. While Roe's flux-difference splitting scheme is effective for high speed flows, it is problematic for nearly incompressible flows with the Mach number $\mathrm{Ma} \ll 1$, because the numerical dissipation is inversely proportional to the local Mach number Ma [20]. Consequently the scheme is too dissipative and thus inaccurate for nearly incompressible flows when the Mach number $\mathrm{Ma} \ll 1$.

In this work we will develop a finite-volume lattice Boltzmann method on arbitrary unstructured meshes with second-order convergence. The gradients at cell centers and the fluxes at cell boundaries are computed by using the least-square method and the low-diffusion Roe scheme, respectively. In addition, the collision term in the LBE is treated implicitly, and the edge-based data structure is adopted in our implementation. These features of the proposed FV-LBM greatly enhance its computational efficiency.

The remainder of the paper is organized as follows. Section 2 describes the finite-volume formulation of the LBE in sufficient details in four parts. Specifically, Section 2.1