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Fast Solution of Three-Dimensional Modified Helmholtz Equations by the Method of Fundamental Solutions

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Abstract. This paper describes an application of the recently developed sparse scheme of the method of fundamental solutions (MFS) for the simulation of three-dimensional modified Helmholtz problems. The solution to the given problems is approximated by a two-step strategy which consists of evaluating the particular solution and the homogeneous solution. The homogeneous solution is approximated by the traditional MFS. The original dense system of the MFS formulation is condensed into a sparse system based on the exponential decay of the fundamental solutions. Hence, the homogeneous solution can be efficiently obtained. The method of particular solutions with polyharmonic spline radial basis functions and the localized method of approximate particular solutions in combination with the Gaussian radial basis function are employed to approximate the particular solution. Three numerical examples including a near singular problem are presented to show the simplicity and effectiveness of this approach.

AMS subject classifications: 65N10, 65N30
Key words: Method of fundamental solutions, exponential decay, modified Helmholtz problem, sparse scheme, localized method of approximate particular solutions, method of particular solutions.

1 Introduction

Helmholtz-type equations arise in many science and engineering aspects which are often used to describe the vibration of a structure, the radiation wave, the acoustic cavity and

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heat conduction problems, just to mention a few [1,2]. Analytical solutions to such problems are difficult or even impossible to obtain whereby the behaviors of the solutions can be understood by numerical methods, such as the finite element method (FEM), the finite difference method (FDM), and the boundary element method (BEM). In spite of the great success of the FEM, the FDM, and the BEM as accurate and effective numerical tools for engineering problems, some disadvantages or inconveniences also perplex the users such as the mesh building especially for three dimensional complex domain problems, numerical quadrature, and singular and hyper-singular integrals [3, 4]. There is still a growing interest in developing new advanced numerical methods, such as meshless or mesh reduction methods in which domain or boundary meshes are reduced or avoided [5–7].

The method of fundamental solutions (MFS) is one of the most powerful meshless techniques that belongs to the category generally named as boundary-type meshless methods which has been applied to many engineering problems, such as acoustic, transient heat conduction, and convection-diffusion problems [8–10]. The MFS is first proposed by Kupradze and Aleksidze [11] which can be classified as the regular BEM. The key idea of the MFS is to represent the solution by a linear combination of fundamental solutions with respect to source points located outside the domain to avoid the singularity of fundamental solutions. Then the problem is transformed into determining the unknown coefficients by requiring approximations to satisfy the given boundary conditions [11, 12]. The MFS has gradually received attentions from science and engineering to solve non-homogeneous problems and various types of time-dependent problems, as long as it is coupled with other techniques which can be used to give an approximation of particular solution, e.g., the method of particular solutions (MPS) [13, 14], the dual reciprocity method (DRM) [15,16], and the multiple reciprocity method (MRM) [17,18].

One of the main drawbacks of the MFS is that the coefficient matrix by the MFS is often dense and ill-conditioned [19–21]. As a result, the traditional MFS is not feasible for solving large scale problems. In the past, the domain decomposition method (DDM), the fast multipole method (FMM), the adaptive cross approximation (ACA), and the matrix decomposition method (MD) [22–25] have been proposed to alleviate storage and computational efficiency problems associated with the MFS formulation. Recently, we propose a new sparse scheme of the MFS by exploiting the exponential decay of the fundamental solution of the modified Helmholtz equation to compress the original dense matrix in order to give a fast solution of two dimensional modified Helmholtz equations [26]. Later, the same idea is extended to establish the fast simulation model of diffusion and wave propagation problems [10].

In this paper, we further extend the idea to give a fast solution of three-dimensional modified Helmholtz problems. The dense coefficient matrix of the MFS formulation is converted into an equivalent sparse system based on the exponential decay of fundamental solutions. As for the particular solution, we have two different strategies, which are, the global and the local approaches. The traditional MPS with polyharmonic radial basis functions (RBFs) which can be viewed as the global method is used to find the particular