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Mechanism behind the Beauty: The Golden Ratio Appeared in the Shape of Red Blood Cells

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Abstract. In the past two decades, a rigorous solution for the shape of human red blood cell (RBC) with a negative spontaneous curvature c_0 has been derived with the Helfrich model under the condition that both the osmotic pressure Δp and tensile stress λ are equal to zero. By fitting the experimentally observed shape of RBC, $c_0 R_0$ has been predicted to be -1.62, the minus golden ratio, where R_0 is the radius of a sphere which has the same surface area as RBC. In this paper, we verify this prediction by comparing experimental data with an analytical equation describing the relation between volume and surface area. Furthermore, it is also found $\rho_{\max}\ /\rho_B \approx 1.6$ with $\rho_{\max}$ the maximal radius and ρ_B the characteristic radius of RBC, showing an approximate beautiful golden cross section of RBC. On the basis of a complete numerical calculation, we propose a mechanism behind the beauty of the minus golden ratio that $c_0 R_0$ results from the balance between the minimization of the surface area and the requirement of adequate deformability of RBC to allow it passing through the spleen, the so called "physical fitness test".

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Key words: Red blood cell, golden ratio.

1 Introduction

Scientists never stop their exploration for red blood cell (RBC) since Jan Swammerdam's pioneer work in 1658 and Anton van Leeuwenhoek's observation of red balls by microscope a few years later [1–4]. In 1992, by applying their theory along with the experimental data provided by Evans [6], Ou-Yang *et al.* [5] found that multiplying RBC's spontaneous curvature by the radius of a sphere with the same surface area of RBC is the negative golden ratio. The study of the golden ratio has been initiated by but not limited

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to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the golden ratio has inspired thinkers of all disciplines like no other numbers in the history of mathematics [7]. In the past 24 years, however, nobody has been able to explain why RBCs adopt the golden ratio in their shape. Our present work is trying to provide a reasonable mechanism to explain it and to predict for other mammalian erythrocytes.

2 Helfrich model

In 1973, Helfrich established the basic model and theory for biological vesicles by introducing a spontaneous curvature to form the following free energy expression [8]

$$F_b=\frac{k}{2}\oint(c_1+c_2-c_0)^2dA+\bar{K}\oint c_1c_2dA, \tag{2.1}$$

where k, $\bar{K}$, c_1,c_2, and c_0 are the bending rigidity, Gaussian curvature modulus, two principal curvatures, and the spontaneous curvature, respectively. c_0 is the size of the membrane vesicle serving as a controlling parameter of the geometry. On the other hand, Kaler *et al.* [9] have reported a general method for producing the well-defined average size of vesicles by using the charge as a controlling parameter. Phenomenologically, we assume that the asymmetry of the membrane induces an electric field $\mathbf{E}=(\psi/d)\mathbf{n}$, where $\psi=\psi^{in}$-ψ^{ext} is the electric potential across the membrane with a thickness of d. Eventually, the relationship between c_0 and ψ yields $c_0=e_{11}\psi/k$, as derived in ref [5]. Ou-Yang and Helfrich derived later in 1987 from Eq. (2.1) a more general equation [10]

$$\Delta p-2\lambda H+k(2H+c_0)(2H^2-2K-c_0H)+2k\nabla^2H=0, \tag{2.2}$$

where H and k are the mean and the Gaussian curvatures, respectively, and ∇^2 is the Laplace-Beltrami operator. Δp and λ are Lagrange multipliers taking into account the constraints that both volume and surface area of vesicles are constants, where Δp = p_o-p_i, is the osmotic pressure difference between the outer and inner media, and λ is the tensile stress. In 1993, Hu *et al.* derived the shape equation for a vesicle with symmetric axes from Eq. (2.2) [11]

$$\begin{aligned}\cos^3\varphi\left(\frac{d^3\varphi}{d\rho^3}\right)=&4\sin\varphi\cos^2\varphi\left(\frac{d^2\varphi}{d\rho^2}\right)\left(\frac{d\varphi}{d\rho}\right)-\cos\varphi\left(\sin^2\varphi-\frac{1}{2}\cos^2\varphi\right)\left(\frac{d\varphi}{d\rho}\right)^3\\&+\frac{7\sin\varphi\cos^2\varphi}{2\rho}\left(\frac{d\varphi}{d\rho}\right)^2-\frac{2\cos^3\varphi}{\rho}\left(\frac{d^2\varphi}{d\rho^2}\right)\\&+\left[\frac{c_0^2}{2}-\frac{2c_0\sin\varphi}{\rho}+\frac{\lambda}{k}-\frac{\sin^2\varphi-2\cos^2\varphi}{2\rho^2}\right]\cos\varphi\left(\frac{d\varphi}{d\rho}\right)\\&+\left[\frac{\Delta p}{k}+\frac{\lambda\sin\varphi}{k\rho}+\frac{c_0^2\sin\varphi}{2\rho}-\frac{\sin^3\varphi+2\sin\varphi\cos^2\varphi}{2\rho^3}\right].\end{aligned} \tag{2.3}$$