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Precorrected-FFT Accelerated Singular Boundary Method for Large-Scale Three-Dimensional Potential Problems

Weiwei Li[1,2], Wen Chen[1,2,*] and Zhuojia Fu[1,2]

[1] *State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, International Center for Simulation Software in Engineering and Sciences, College of Mechanics and Materials, Hohai University, Nanjing 210098, P.R. China.*
[2] *State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, P.R. China.*

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Abstract. This study makes the first attempt to accelerate the singular boundary method (SBM) by the precorrected-FFT (PFFT) for large-scale three-dimensional potential problems. The SBM with the GMRES solver requires $\mathcal{O}(N^2)$ computational complexity, where N is the number of the unknowns. To speed up the SBM, the PFFT is employed to accelerate the SBM matrix-vector multiplication at each iteration step of the GMRES. Consequently, the computational complexity can be reduced to $\mathcal{O}(N\log N)$. Several numerical examples are presented to validate the developed PFFT accelerated SBM (PFFT-SBM) scheme, and the results are compared with those of the SBM without the PFFT and the analytical solutions. It is clearly found that the present PFFT-SBM is very efficient and suitable for 3D large-scale potential problems.

AMS subject classifications: 65M70, 65N35, 65T50

Key words: Singular boundary method, precorrected-FFT, fast matrix algorithm, three-dimensional potential problems, large-scale.

1 Introduction

The singular boundary method (SBM) [1] is a recent strong-form technique free of numerical integration to solve the boundary value problems. As a boundary-type meshless method, it can obtain the same accuracy of the boundary element method (BEM) without the mesh-generation. Since the original intensity factors (OIFs) are introduced, the singularities are removed [2]. This method can be viewed as a modified version of the method of fundamental solutions (MFS). However, unlike the MFS, the SBM distributes

*Corresponding author. *Email addresses:* liweiwei_hhu@163.com (W. Li), chenwen@hhu.edu.cn (W. Chen), paul212063@hhu.edu.cn (Z. Fu)

the source points coinciding with the collocation points on the physical boundary to avoid the perplexing fictitious boundary issue. Therefore, the SBM is truly meshless, free of integration, and easy-to-implement. Due to these advantages, the SBM is considered a competitive alternative method for some certain problems.

However, the SBM will run into difficulties when large-scale problems are encountered. Like in the BEM, the SBM generates a dense $N\times N$ resultant interpolation matrix, where N is the number of the unknowns [3]. Computing this kind of matrix equation is computationally very expensive and requires $\mathcal{O}(N^2)$ operations. Otherwise, if the direct solver is used, for solving the system of equations, $\mathcal{O}(N^3)$ operations are required. Thanks to costly CPU time and huge memory demands, the standard SBM is largely restricted to the small and medium-scale applications.

The acceleration techniques, when combined with the iterative linear system solver GMRES [4], is an effective strategy to speed up the SBM as in the case of the BEM. Recently, the fast multipole method (FMM) [5, 6] has been successfully applied to the SBM solution of potential [3] and Helmholtz problems [7].

As an alternative accelerating technique, this paper makes the first attempt to present the development of a precorrected-FFT (PFFT) [8] accelerated SBM approach for large-scale three-dimensional (3D) potential problems. The PFFT technique is at best an $\mathcal{O}(N\log N)$ algorithm, and the advantage of the PFFT is that it is relatively kernel independent [9], and easier to implement. It has been successfully applied to electromechanical systems [10–12], elastodynamics [9, 13], composite materials [14, 15], nonlinear wave dynamics [16, 17], acoustic problems [18, 19], just to mention a few.

The rest of this paper is organized as follows. We describe the SBM formulation for 3D potential problems followed by coupling of the SBM with the PFFT. Next, three benchmark examples are investigated to valid the developed the PFFT accelerated SBM (PFFT-SBM) strategy. The PFFT-SBM is compared with those of the SBM without the PFFT and the analytical solutions. The conclusions are drawn lastly.

2 The SBM formulation for 3D potential problems

Consider the Laplace equation governing 3D potential problems

$$\nabla^2 u(x)=0, \quad x\in\Omega, \tag{2.1}$$

subjected to boundary conditions

$$u(x)=\bar{u}(x), \quad x\in\Gamma_D, \tag{2.2}$$

$$q(x)=\frac{\partial u}{\partial n}(x)=\bar{q}(x), \quad x\in\Gamma_N, \tag{2.3}$$

where u is the potential field, ∇^2 represents the Laplacian operator, n indicates the unit outward normal, and $\Gamma=\Gamma_D\cup\Gamma_N=\partial\Omega$ denotes the whole boundary of the domain Ω.