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On 16th and 32th Order Multioperators-Based Schemes for Smooth and Discontinuous Fluid Dynamics Solutions

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Abstract. The paper presents a novel family of arbitrary high order multioperators approximations for convection, convection-diffusion or the fluid dynamics equations. As particular cases, the 16th- and 32th-order skew-symmetric multioperators for derivatives supplied by the 15th- and 31th-order dissipation multioperators are described. Their spectral properties and the comparative efficiency of the related schemes in the case of smooth solutions are outlined. The ability of the constructed conservative schemes to deal with discontinuous solutions is investigated. Several types of nonlinear hybrid schemes are suggested and tested against benchmark problems.

AMS subject classifications: 65M06, 65M99, 76L05, 76L99

Key words: Arbitrary-order discretization, multioperators, equations with convection terms, 16th- and 32th-order schemes, discontinuous solutions.

1 Introduction

The concept of multioperators (that is, linear combinations of basis grid operators) introduced by the author in [1] is aimed at constructing prescribed-order numerical analysis formulas (formally, arbitrary-order ones).

The multioperators idea is in line with the general trend of constructing high order methods for computational fluid dynamics (CFD) applications. The well known approaches are the efficient spatial discretizations of the Euler equations based on the centered compact approximations [3] as well as on the Weighted Essentially Non-Oscillatory (WENO) principle admitting enlarged stencils [5,6]. In contrast to the above methods, the ADER schemes [7, 8] allow to create arbitrary high order approximations to hyperbolic systems both in space and time.

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In the CFD context, the multioperators approach may be viewed as an alternative to the commonly used ways of increasing approximation orders of the spatial discretizations. Sketching the broad outlines, to increase the orders in the traditional way, one increases numbers of *basis functions* defining underlining polynomials. In contrast, an increase of the orders via multioperators is accomplished by increasing numbers of *basis operators*.

Several types of multioperators were constructed during recent years, primary emphasis being placed on CFD applications. Their main features were described for example, in [2,9]. Initially, the basis operators were obtained using compact upwind differencing (CUD) operators approximating first derivatives [10]. The three-diagonal inversions were needed to calculate the multioperators actions on known grid functions. The next generation of the type reported in [9] requires inversions of two-point operators thus considerably reducing the operation counts. In all cases, the basis operators make up upwind-downwind pairs.

Mathematically, it means that the pairs are composed by the operators with the same skew-symmetric components. Their self-adjoint components are positive or negative operators differing from one another only by their signs. The corresponding multioperators also make up upwind-downwind pairs provided that parameters values were chosen properly. The pairs were used to approximate convection terms in the flux splitting manner. As a result, the positive self-adjoint components of the multioperators serve as high-order dissipation mechanisms which spectral content can not be controlled independently of the skew-symmetric components.

In the present paper, another way of using upwind-downwind pairs is suggested. Instead of constructing upwind-downwind multioperators, the skew-symmetric multioperators which basis operators are generated by the half-sum of the operators from the pair. It provides high-order dissipation-free approximation to first derivatives. The difference of those operators serves as a generator of the basis operators for the self-adjoint positive multioperator which can be used if high-order dissipation is needed. The basis operators of the latter can be defined by other values of the parameter, their number being possibly different from that for the first one.

The above approach possesses the following merits.

Firstly, higher orders can be obtained with the same numbers of the parameter's values since either even or odd powers of mesh sizes are annihilated in the corresponding truncation errors.

Secondly, the linear systems for the coefficients of the multioperators can be practically well-conditioned when increasing numbers of the basis operators. It allows to construct extremely high-order approximations to derivatives. In contrast, the matrices of the systems in the cases of upwind or downwind basis operators are usually of the Vandermonde type which means the possibility of encountering ill-conditioned systems for high-order multioperators.

Finally, the spectral content of the dissipation multioperator can be efficiently controlled independently of the skew-symmetric one.