

Comparison of Finite Difference and Mixed Finite Element Methods for Perfectly Matched Layer Models

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Abstract. We consider the anisotropic uniaxial formulation of the perfectly matched layer (UPML) model for Maxwell's equations in the time domain. We present and analyze a mixed finite element method for the discretization of the UPML in the time domain to simulate wave propagation on unbounded domains in two dimensions. On rectangles the spatial discretization uses bilinear finite elements for the electric field and the lowest order Raviart-Thomas divergence conforming elements for the magnetic field. We use a centered finite difference method for the time discretization. We compare the finite element technique presented to the finite difference time domain method (FDTD) via a numerical reflection coefficient analysis. We derive the numerical reflection coefficient for the case of a semi-infinite PML layer to show consistency between the numerical and continuous models, and in the case of a finite PML to study the effects of terminating the absorbing layer. Finally, we demonstrate the effectiveness of the mixed finite element scheme for the UPML by a numerical example and provide comparisons with the split field PML discretized by the FDTD method. In conclusion, we observe that the mixed finite element scheme for the UPML model has absorbing properties that are comparable to the FDTD method.

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1 Introduction

The effective modeling of electromagnetic waves on unbounded domains by numerical techniques, such as the finite difference or the finite element method, is dependent on the particular absorbing boundary condition used to truncate the computational domain. In 1994, J. P. Berenger created the *perfectly matched layer* (PML) technique for the reflectionless absorption of electromagnetic waves in the time domain [4]. The PML is

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an absorbing layer that is placed around the computational domain of interest in order to attenuate outgoing radiation. Berenger showed that his PML model allowed perfect transmission of electromagnetic waves across the interface of the computational domain regardless of the frequency, polarization or angle of incidence of the waves. The waves are then attenuated exponentially with respect to depth into the absorbing layers. Since its original inception in 1994, PML's have also extended their applicability in areas other than computational electromagnetics such as acoustics, elasticity, etc., [2, 3, 15–17].

The properties of the continuous PML model have been studied extensively and are well documented. The original *split field* PML, proposed by Berenger, involved a nonphysical splitting of Maxwell's equations resulting in non-Maxwellian fields and a weakly hyperbolic system [1]. A complex change of variables approach was used in [9,20] to derive an equivalent PML model that did not require a splitting of Maxwell's equations. In [22] the authors observed that a material can possess reflectionless properties if it is assumed to be anisotropic. A single layer in this technique was termed *uniaxial*, and the PML was referred to as the uniaxial PML (UPML). In this method, modifications to Maxwell's equations are also not required and one obtains a strongly hyperbolic system. In [14,18] further study of the anisotropic PML is carried out. Unlike Berenger's split field PML, which is a nonphysical medium, the anisotropic PML can be a physically realizable medium [20]. Thus, there are several reasons for using the anisotropic PML in numerical simulations. In [24] the authors show that the anisotropic PML and Berenger's split field PML produce the same tangential fields; however, the normal fields are different as the two methods satisfy different divergence conditions.

The finite depth of the absorbing layer allows the transmitted part of the wave to return to the computational domain. In addition, the discretization of Maxwell's equations introduces errors which cause the PML to be less than perfectly matched. Even so, it has been found that the PML medium can result in reflection errors as minute as -80 dB to -100 dB [4, 5, 9, 14].

There are a number of publications that study the properties of the finite difference time domain (FDTD) method (Yee scheme [26]) for discretizing the PML model (e.g., see [23]). There are significantly fewer publications that study the properties of the finite element method for the approximation of the PML equations. A comparison of the anisotropic PML to the split field PML of Berenger was performed in [24], in which the authors implement the anisotropic PML into an edge based finite element method for a second order formulation of Maxwell's equations. In [25] the authors use the lowest order as well as first order tangential vector finite element methods for the discretization of the electric field. They compare the performance of these elements with the FDTD method when a PML is used to terminate the computational domain. They show that the lowest order elements do not perform as well as the FDTD method; however, the first order elements can produce more accurate results than FDTD. A time domain mixed finite element method has been used in [11] along with mass lumping techniques to solve scattering problems on domains where a PML method based on the Zhao-Cangellaris's model is used to terminate the mesh [27]. The underlying partial differential equations in