

Wave Propagation Simulation Using the CIP Method of Characteristic Equations

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Abstract. We apply the CIP (Cubic Interpolated Profile) scheme to the numerical simulation of the acoustic wave propagation based on characteristic equations. The CIP scheme is based on a concept that both the wavefield and its spatial derivative propagate along the same characteristic curves derived from a hyperbolic differential equation. We describe the derivation of the characteristic equations for the acoustic waves from the basic equations by means of the directional splitting and the diagonalization of the coefficient matrix, and establish geophysical boundary conditions. Since the CIP scheme calculates both the wavefield and its spatial derivatives, it is easy to realize the boundary conditions theoretically. We also show some numerical simulation examples and the CIP can simulate acoustic wave propagation with high stability and less numerical dispersion. The method of characteristics with the CIP scheme is a very powerful technique to deal with the wave propagation in complex geophysical problems.

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1 Introduction

The Finite Difference method has been widely accepted for wave simulation in solids because of the simplicity of computational implementation. However, careful treatment of the numerical dispersion and computational stability are required in order to obtain the meaningful simulation results. Several advanced techniques are known to reduce numerical dispersion, for example; staggered grids scheme [8] and the high order computation scheme [5]. The pseudo spectra method [3] is also well known as a highly accurate numerical scheme in the Fourier domain. The CIP (Cubic Interpolated Profile)

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scheme [10, 11] was proposed as a stable and less dispersive scheme in CFD (Computational Fluid Dynamics) and applied to many difficult problems such as the plasma phenomena [4, 11]. The CIP method in combination with the method of characteristics was developed to simulate the Maxwell equation accurately, and it was compared with FDTD method in [6]. The CIP scheme is based on a fact that not only the wavefield but also its spatial derivatives propagate along the same characteristic curve derived from a hyperbolic differential equation.

In this paper we apply the CIP scheme to simulate the P wave propagation by solving an acoustic wave equation. We derive the characteristic equations for the acoustic wave and we solve these equations by the CIP scheme, and establish the treatment of several geophysical boundary conditions such as; the free surface boundary, the irregular topographic boundary, and the absorbing boundary. We also show results of numerical simulations for a simple half space model and topographic variation model.

2 CIP scheme

The phenomenon of the wave propagation in one dimensional space obeys the following first-order differential equation,

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = 0. \quad (2.1)$$

This first-order wave equation shows that a wave packet on the wavefield f propagates along a curve $dx/dt = u$ in the phase space. This curve is known as a characteristic curve and Eq. (2.1) is called a characteristic equation for the forward propagation of the wavefield. Although this equation is simple, it is difficult to evaluate numerically with high stability and less numerical dispersion. The CIP scheme can overcome these problems by solving not only (2.1) but also a differential equation for a spatial derivative of the wavefield f . If the propagation velocity u is constant, we obtain the same equation as (2.1) for g which is a spatial derivative of f [10],

$$\frac{\partial g}{\partial t} + u \frac{\partial g}{\partial x} = 0, \quad g = \frac{\partial f}{\partial x}. \quad (2.2)$$

These two equations, (2.1) and (2.2), become the governing equations for the propagation of the wavefield f and its spatial derivatives g obey the same characteristic equations. The CIP scheme utilize this property in solving a hyperbolic differential equation and Fig. 1 shows conceptual diagrams of the CIP scheme. In Fig. 1(a), the solid line corresponds to an initial wave packet and dashed line becomes an exact solution at one time-step ahead. Solving the wave equation numerically using the finite difference approximation, we may obtain the white circle (see in Fig. 1(a)) after one time progressed. If the values of the wavefield between the grid points are interpolated linearly using values at each grid, the numerical diffusion occurs shown in Fig. 1(b). However, if we can use the information of the spatial derivatives at every grid points, we can overcome this numerical dispersion problem and may keep the original shape of the wave packet through