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# Forward Scattering Series for 2-Parameter Acoustic Media: Analysis and Implications to the Inverse Scattering Task Specific Subseries

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**Abstract.** We study the 2-parameter acoustic Born series for an actual medium with constant velocity and a density distribution. Using a homogeneous background we define a perturbation, the difference between actual and reference medium (we use background and reference as synonyms), which exhibits an anisotropic behavior due to the density distribution. For an actual medium with a constant velocity, the reference velocity can be selected so that the waves in the actual medium travel with the same speed as the waves in the background medium. Scattering theory decomposes the actual wave field into an infinite series where each term contains the perturbation and the propagators in the background medium. Hence, in this formalism, all propagations occur in the background medium and the actual medium is included only through the perturbations which scatter the propagating waves. The density-only perturbation has an isotropic and an anisotropic component. The anisotropic component is dependent on the incident direction of the propagating waves and behaves as a *purposeful perturbation* in the sense that it annihilates the part of the Born series that acts to correct the time to build the actual wave field, an unnecessary activity when the reference velocity is equal to the one in the actual medium. This means that the forward series is not attempting to correct for an issue that does not exist. We define the purposeful perturbation concept as the intrinsic knowledge of precisely what a given term is designed to accomplish. This is a remarkable behavior for a formalism that predicts the scattered wave field with an infinite series. At each order of approximation the output of the series is consistent with the fact that the time is correct because the velocity is always constant. In the density-only perturbation, the forward series only seeks to predict the correct amplitudes. Finally, we extend the analysis to a wave propagating in a medium where both density and velocity change. By selecting a convenient set of parameters, we find a conceptual framework for the multiparameter Born series. This framework provides an insightful analysis that can be mapped and applied to the concepts and algorithms of the inverse scattering series.

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# 1 Introduction

The inverse scattering series has proven to be a good framework for solving the free surface and internal multiple problem [17, 18] without the need for a velocity model. Recently, the inverse scattering series has given results which indicate that it is also a good framework for doing imaging and inversion without a velocity model [5,6,13,14,19, 21]. In [13] it is shown that a 1D earth can be imaged without the velocity model. Later, [6] showed some early examples where the inverse scattering series is used to image a 2D earth without the velocity model.

In the above references, it is assumed that the inverse scattering series can be divided into different subseries, where each subseries is responsible for solving a single task of the inverse problem [19]. These single tasks are divided into the tasks which are common in a standard seismic processing workflow: 1) Free surface multiple removal, 2) Internal multiple removal, 3) Imaging, and 4) Inversion for earth parameters. In addition, the inverse scattering series will contain terms that contribute to solving more than a single task. These terms are omitted in the framework suggested in [19].

In this paper we study the forward scattering series (also known as Born or Neumann series) in order to identify or shed light on which terms in the inverse scattering series are important for performing imaging and inversion. In [7] it is shown analytically for the 1D one parameter acoustic wave equation the validity of the ideas and concepts introduced in [17–19] and used in the development of inverse scattering processing methods (see for example [19]). The mathematical analysis and study of the forward series and its relation with seismic events was revisited by [3,4,9–11]. In [3] absorption and velocity changes in the transmission analysis of the forward series were included. In [4] modeling of specific events (primaries, multiples, diffractions) with analytic, wave-theoretic expressions derived from the forward scattering series in complex 3D scalar media with only velocity changes were proposed. Padé approximants to improve the efficiency of the forward series when used to model acoustic wave field propagation in a vertically varying medium with constant density was introduced in [10,11]. We will extend the analysis in [7, 8] by studying how the forward scattering series builds up the solution of the two parameter acoustic wave equation from a homogenous background. We will find that the terms containing a velocity perturbation are the terms that contribute to the construction of the actual travel-time for the wave propagating through the inhomogeneous medium. The density perturbation will only contribute to building up the correct amplitude response of the actual wave field.

In Section 2, we introduce the Lippmannn-Schwinger equation and the forward scattering series constructed from the two-parameter acoustic wave equation. It also gives