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Forward Scattering Series and Padé Approximants for Acoustic Wavefield Propagation in a Vertically Varying Medium

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Abstract. We present the application of the theory of Padé approximants to extending the perturbative solutions of acoustic wave equation for a three dimensional vertically varying medium with one interface. These type of solutions have limited convergence properties depending on either the degree of contrast between the actual and the reference medium or the angle of incidence of a plane wave component. We show that the sequence of Padé approximants to the partial sums in the forward scattering series for the 3D wave equation is convergent for any contrast and any incidence angle. This allows the construction of any reflected waves including phase-shifted post-critical plane waves and, for a point-source problem, refracted events or headwaves, and it also provides interesting interpretations of these solutions in the scattering theory formalism.

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Key words: Acoustic wave modeling, forward scattering series, Padé approximants.

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1 Introduction

Many fields of non-destructive evaluation of a medium properties involve an acoustic, elastic or electro-magnetic experiment in which a natural or artificially created wave propagates through that medium and is recorded outside of the medium. The goal in such an experiment is to process the recorded wave, the data, to determine the medium's internal structure (imaging) and properties (inversion). Examples of such fields of applications are geophysical exploration for natural resources, medical imaging, remote sensing in engineering, whole earth seismology, astronomy, military radar and underground object detection, etc. Their tremendous economical, social and military importance is evident.

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In order to extract meaningful and useful information from the recorded wave-field, one needs to predict or model how the wave-field propagates in different types of complex settings. Ideally, one would be able to solve exactly the differential equations describing the propagation of the wave-field and add boundary conditions to describe different structures embedded in that medium. The solutions of such a boundary value problem would characterize the wave propagation through the medium, and its interactions with the complex structures it encounters, and would construct the wave-field everywhere inside and outside the medium. Unfortunately, such exact analytical solutions are difficult (and most often impossible) to obtain. This lead to the development of alternative methods, for example numerical, for modeling the propagation of the wave-field in complex realistic settings; from these we mention least squares [12], finite differences [2,3,21], ray tracing [8], Fourier or pseudo-spectral methods [26], finite elements [9], reflectivity [15] and [31] as well as other hybrid methods [14]. A compilation of classical papers describing the finite differences and finite element methods in geophysics, their accuracy and different types of boundary conditions appeared in [22].

Many other methods, either new or derived from these ones, have been developed and implemented, each one trying to address a specific issue or downside in the methods listed above. For example the modeling techniques based on Kirchhoff integral [16], $f-k$ solutions to the wave equation [39], paraxial extrapolators method [10], Gaussian beam methods [24, 25], hyperbolic superposition [27], scattering theory [42], lattice Boltzmann method [17] just to mention a few of them. All these methods have different assumptions, strengths and limitations. One feature shared by all the modeling methods, is that, as the complexity of the geological models increases, the computational requirements also increase to very expensive, and sometimes prohibited, values. Often a lower and cheaper alternative is chosen to model the wave-field in the detriment of accuracy. New methods and alternatives are sought and developed every year (see e.g. the Seismic Modeling sections at the American Geophysical Union (AGU) and Society for Exploration Geophysicists (SEG) annual meetings) to address some of the difficulties, execution time and computational costs in modeling wave-field propagation in complex sub-surface conditions.

In this paper we discuss a recently developed tool, for modeling the propagation of seismic wavefields, based on the scattering theory, the forward scattering series. Scattering theory is a powerful and useful method for analyzing wave propagation in a given medium (see e.g. [32, 42]). As any form of perturbation theory, it relates the propagation of the wave in that medium with the propagation of the wave in a reference medium and a perturbation operator which describes the difference between the two media. The forward problem is to construct the actual wave-field everywhere given the reference wave-field and the perturbation operator; the inverse problem is to construct the perturbation operator (and hence the unknown medium) given the reference wave-field everywhere and the actual wave-field on a measurement surface outside the unknown medium (data). This relation between the three quantities is nonlinear and, to date, it can only be represented using the Born or Neumann series (see e.g. [36]), which, when con-