REVIEW ARTICLE

Spectral-Element and Adjoint Methods in Seismology

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Abstract. We provide an introduction to the use of the spectral-element method (SEM) in seismology. Following a brief review of the basic equations that govern seismic wave propagation, we discuss in some detail how these equations may be solved numerically based upon the SEM to address the *forward problem* in seismology. Examples of synthetic seismograms calculated based upon the SEM are compared to data recorded by the Global Seismographic Network. Finally, we discuss the challenge of using the remaining differences between the data and the synthetic seismograms to constrain better Earth models and source descriptions. This leads naturally to adjoint methods, which provide a practical approach to this formidable computational challenge and enables seismologists to tackle the *inverse problem*.

Contents

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1

1   Introduction

The spectral-element method (SEM) has been used for more than two decades in computational fluid dynamics [48], but it has only recently gained popularity in seismology. Initially the method was applied to 2D seismic wave propagation problems [15,50], but currently the SEM is widely used for 3D regional [21,28,29,33,37,55] and global [10–12,30,31,35,36] seismic wave propagation. Recent reviews of the SEM in seismology may be found in [38] and [13].

Like a classical finite-element method, the SEM is based upon an integral or weak implementation of the equation of motion. It combines the accuracy of the global pseudospectral method with the flexibility of the finite-element method. The wavefield is typically represented in terms of high-degree Lagrange interpolants, and integrals are computed based upon Gauss-Lobatto-Legendre quadrature, which leads to a simple explicit time scheme that lends itself very well to calculations on parallel computers.

The current challenge lies in harnessing these numerical capabilities to enhance the quality of tomographic images of the Earth's interior, in conjunction with improving models of the rupture process during an earthquake. [62] demonstrated that this problem may be solved iteratively by numerically calculating the derivative of a waveform misfit function. The construction of this derivative involves the interaction between the wavefield for the current model and a wavefield obtained by using the time-reversed waveform differences between the data and the current synthetics as simultaneous sources. Only two numerical simulations are required to calculate the gradient of the misfit function: one for the current model and a second for the time-reversed differences between the data and the current synthetics. [60] generalized the calculation of the derivative of a misfit function by introducing the concept of an 'adjoint' calculation. The acoustic theory developed by [62] was extended to the anelastic wave equation by [63,64]. Applications of the theory may be found in [1,2,17,23,45,46,49,61].

The purpose of this article is to review the use of the SEM in seismology, and to illustrate the powerful combination of the SEM for the *forward problem*, i.e., given a 3D Earth model and a (finite) source model accurately simulate the associated ground motions, with adjoint methods for the *inverse problem*, i.e., using the remaining differences between the data and the simulations to improve source and Earth models.

2   Basic theory of seismology

2.1   Earth models

Seismologists have determined the average, spherically symmetric structure of the Earth with a high degree of accuracy. A typical one-dimensional (1D) model is the Preliminary Reference Earth Model (PREM) [20], shown in Fig. 1. Such an isotropic, elastic Earth model may be characterized in terms of three parameters: the distribution of mass density ρ, the compressional wave speed α, and the shear wave speed β. Rather than the