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Retrieval of Elastic Green's Tensor near a Cylindrical Inhomogeneity from Vector Correlations

Juan A. Pérez-Ruiz[1,*], Francisco Luzón[1] and Francisco J. Sánchez-Sesma[2]

[1] *Grupo de Geofísica Aplicada, Departamento de Física Aplicada, Universidad de Almería, Cañada de San Urbano s/n, 04120-Almería, Spain.*
[2] *Departamento de Sismología, Instituto de Ingenieria, Universidad Nacional Autónoma de México, Cd. Universitaria, Circuito Escolar s/n, Coyoacán 04510, México D. F., Mexico.*

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Abstract. Multiple scattering of elastic waves in realistic media makes that average field intensities or energy densities follow diffusive processes. In such regime the successive P to S energy conversions by distributed random inhomogeneities give rise to equipartition which means that in the phase space the available elastic energy is distributed in average with equal amounts among the possible states of P and S waves. In such diffusive regime the P to S energy ratio equilibrates in an universal way independent of the particular details of the scattering. It has been demonstrated that averaging the cross correlations at any two points of an elastic medium subjected to diffuse elastic wavefields leads to the emergence of the Green function, which is the wave field that would be observed at one position if an impulsive load is applied at the other. In this work we study the problem of the retrieval of the 2D tensor elastodynamic Green function in an infinite elastic space containing a circular cylinder inclusion. We illuminate isotropically the elastic space with plane waves. We assume the spectra for both P and S waves uniform but such that the energy ratio $E_S/E_P=(\alpha/\beta)^2$, which is the one predicted by equipartition theory in two-dimensions. We then show that the Fourier transform of azimuthal average of the cross-correlation of motion between two points within an elastic medium is proportional to the imaginary part of the exact Green tensor function between these points. The numerical results presented here point out the possibility of detection and imaging of diffractors and resonant diffractors by cross correlation even in presence of attenuation exists.

AMS subject classifications: 86A15, 74J20, 74B05

Key words: Multiple scattering, elastic waves, elastic Green's tensor.

*Corresponding author. *Email addresses:* japerez@ual.es (J. A. Pérez-Ruiz), fluzon@ual.es (F. Luzón), sesma@servidor.unam.mx (F. J. Sánchez-Sesma)

1 Introduction

The use of correlations of seismic noise and coda waves is becoming a subject of interest as it is now becoming clear that there is valuable information in these waves. We may track various previous developments that lead to the concept of diffuse wave fields. However, the pioneering approach of Aki (1957) is without any doubt crucial to understand the roles of seismic noise and coda. Aki studied descriptions that ranged from single and multiple scattering to radiative transfer ideas that he explored in order to explain coda envelopes (see Sato and Fehler, 1998).

The elastodynamic Green function has been recovered from the averaging of cross correlation of the isotropic elastic wavefield generated by either multiple scattering or by a large number of sources (such as microseisms) or microtemor as well (see Campillo and Paul, 2003; Shapiro and Campillo, 2004, Sabra et al., 2005; Shapiro et al., 2005, Chávez-García and Luzón, 2005). These experimental results have demonstrated the role of long range correlation. The Green function between two points is the wave field that would be observed at one position if an impulsive load is applied at the other. The accuracy of the reconstructed Green function depends critically on the duration of the signals processed. Theoretically the cross-correlations should be applied to equipartitioned fields (that are in a diffusive regime in which the net energy flux is null). This takes place after sufficiently long time to allow multiple scattering (and thus diffusion) of the wave field. Equipartition means that in the phase space the available energy is equally distributed, with fixed average amounts, among all the possible states. Extending these ideas of thermodynamics, equipartition has been introduced in acoustics and elastic wave propagation. When multiple scattering take place the energy ratios of the various modes tends to stabilize to a constant value, independent of the details of the scattering (see Ryzhik et al., 1996). The ratio of S and P energies in the equipartition regime ratio for the full elastic space in 2D and 3D has been obtained by Weaver (1982). It can be obtained using different arguments (see the Appendix in Sánchez-Sesma and Campillo, 2006).

In a recent work (Sánchez-Sesma and Campillo, 2006) the case of the full homogeneous elastic medium was studied both in 2D and 3D. In such study isotropic illumination and equipartition was assumed and it was demonstrated that the Fourier transform of the azimuthal average of the cross-correlation between the vector motions at two points within an infinite elastic space is proportional to the imaginary part of the exact Green tensor function between these points. This elastic case shows that both equipartition and isotropy of the field are necessary conditions to retrieve the exact Green function from correlations of the elastic field.

For horizontally layered medium Claerbout (1968) showed that the autocorrelation of the transmission response leads to reflection seismograms together with their time-reversed part. The source in the underground may be transient or noisy. In any case, the source signature in the reconstructed surface response is related to the autocorrelation of the source signal. Claerbout derivation was one dimensional but the idea has been applied to micro earthquake data (Daneshvar et al., 1995).