

## Boundary Integral Modelling of Elastic Wave Propagation in Multi-Layered 2D Media with Irregular Interfaces

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**Abstract.** We present a semi-analytic method based on the propagation matrix formulation of indirect boundary element method to compute response of elastic (and acoustic) waves in multi-layered media with irregular interfaces. The method works recursively starting from the top-most free surface at which a stress-free boundary condition is applied, and the displacement-stress boundary conditions are then subsequently applied at each interface. The basic idea behind this method is the matrix formulation of the propagation matrix (PM) or more recently the reflectivity method as wide used in the geophysics community for the computation of synthetic seismograms in stratified media. The reflected and transmitted wave-fields between arbitrary shapes of layers can be computed using the indirect boundary element method (BEM, sometimes called IBEM). Like any standard BEM, the primary task of the BEM-based propagation matrix method (thereafter called PM-BEM) is the evaluation of element boundary integral of the Green's function, for which there are standard method that can be adapted. In addition, effective absorbing boundary conditions as used in the finite difference numerical method is adapted in our implementation to suppress the spurious arrivals from the artificial boundaries due to limited model space. To our knowledge, such implementation has not appeared in the literature. We present several examples in this paper to demonstrate the effectiveness of this proposed PM-BEM for modelling elastic waves in media with complex structure.

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## 1 Introduction

Computation of elastic wave propagation in layered media with arbitrary shapes of interfaces has found applications in many areas, such as engineering, geophysics, underwater acoustics, etc. Traditionally, domain-based finite difference or finite element methods are used. For stratified flat layered media, the propagation matrix or more recently reflectivity method can be used (Fuchs and Muller, 1971). Chen (1990, 1995, 1996) has extended the propagation matrix method to multi-layered media with irregular interfaces using the so-called global generalized reflection/transmission formulations. Recently the indirect Boundary Element Method (BEM) has been extended to model wave propagation in multi-layered media with arbitrary interfaces (see for examples, Bouchon et al., 1989; Bouchon & Coutant, 1994; Pedersen et al., 1996, Vai et al., 1999, and cited references in those papers). Note that BEM has been extensively used to study topographic effects using exact Green functions by Sanchez-Sesma and his co-authors (e.g. Sanchez-Sesma and Campillo, 1991). This method has a distinct advantage over domain-based methods in that only boundaries, or in the case of multi-layers interfaces, need to be discretized. This method is based on the matrix formulation of the propagation matrix method (Fuchs and Muller, 1971; Kennett, 1981) and essentially works recursively to match boundary conditions at each successive interface (Pedersen et al., 1996). Reflection and transmission at internal interfaces are computed using the BEM. We shall refer to this method as PM-BEM. We have tested extensively the validity and limitations of the PM-BEM and its stability in a variety of situations, examining in particular dependence on source frequency, distance of the source from boundaries and separation of two boundaries. Comparison with results from the reflectivity method shows that this PM-BEM is very accurate. The method can be potentially used to perform large scale seismic modelling.

## 2 Indirect boundary element method

For simplicity, we shall consider 2D here. In the absence of body forces the displacement  $\vec{u}$  at any point  $\vec{x}$  in an area  $V$  surrounded by the boundary  $S$  (Fig. 1) can be expressed as follow, i.e. mathematical description of Huygen's principle (Liu et al., 1997; Liu & Zhang, 2001; Pointer et al., 1998):

$$u_i(\vec{x}) = \int_S \varphi_j(\vec{x}') G_{ij}(\vec{x}, \vec{x}') dS', \quad (2.1)$$

where  $\vec{G}_{ij}(\vec{x}, \vec{x}')$  is the  $i$ th displacement Green's function at  $\vec{x}$  due to a point source in  $j$ th direction at  $\vec{x}'$  (the variable with an arrow above implies a vector, matrix or tensor).  $\varphi_j(\vec{x}')$  is the force density at  $\vec{x}'$  in  $j$ th direction.

The corresponding expression for traction, for a smooth boundary, is given by:

$$\tau_i(\vec{x}) = c\varphi_i(\vec{x}) + \int_S \varphi_j(\vec{x}') T_{ij}(\vec{x}, \vec{x}') dS', \quad (2.2)$$