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A Comparative Study of LBE and DUGKS Methods for Nearly Incompressible Flows

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Abstract. The lattice Boltzmann equation (LBE) methods (both LBGK and MRT) and the discrete unified gas-kinetic scheme (DUGKS) are both derived from the Boltzmann equation, but with different consideration in their algorithm construction. With the same numerical discretization in the particle velocity space, the distinctive modeling of these methods in the update of gas distribution function may introduce differences in the computational results. In order to quantitatively evaluate the performance of these methods in terms of accuracy, stability, and efficiency, in this paper we test LBGK, MRT, and DUGKS in two-dimensional cavity flow and the flow over a square cylinder, respectively. The results for both cases are validated against benchmark solutions. The numerical comparison shows that, with sufficient mesh resolution, the LBE and DUGKS methods yield qualitatively similar results in both test cases. With identical mesh resolutions in both physical and particle velocity space, the LBE methods are more efficient than the DUGKS due to the additional particle collision modeling in DUGKS. But, the DUGKS is more robust and accurate than the LBE methods in most test conditions. Particularly, for the unsteady flow over a square cylinder at Reynolds number 300, with the same mesh resolution it is surprisingly observed that the DUGKS can capture the physical multi-frequency vortex shedding phenomena while the LBGK and MRT fail to get that. Furthermore, the DUGKS is a finite volume method and its computational efficiency can be much improved when a non-uniform mesh in the physical space is adopted. The comparison in this paper clearly demonstrates the progressive improvement of the lattice Boltzmann methods from LBGK, to MRT, up to the current DUGKS, along with the inclusion of more reliable physical process in their algorithm development. Besides presenting the Navier-Stokes solution, the DUGKS can capture the rarefied flow phenomena as well with the increasing of Knudsen number.

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1 Introduction

In recent years, the development of Boltzmann equation-based kinetic schemes has received particular attentions due to their distinctive modeling for flow simulations. The distinctive features in the kinetic methods include the following two aspects. Firstly, the Boltzmann equation provides a theoretical foundation for the hydrodynamic description from the underlying microscopic physics. Besides capturing the Navier-Stokes (NS) solutions, the kinetic methods can be used to study non-equilibrium flows in the transition regime. Secondly, the Boltzmann equation is a first-order integro-partial-differential equation with a linear advection term, while the Navier-Stokes equations are second-order partial differential equations with a nonlinear advection term. The nonlinearity in the Boltzmann equation resides in its collision term, which is local. Therefore, the kinetic equation is more feasible to handle the discontinuities or unresolved flow regions. This feature may lead to some computational advantages for computational fluid dynamics (CFD) [1]. Due to the mesoscopic nature, kinetic methods are particularly appealing in modeling and simulating complex and non-equilibrium flows [2].

There have been a number of kinetic or mesoscopic methods, such as the lattice gas cellular automata (LGCA) [3], the lattice Boltzmann equation (LBE) [4], the gas-kinetic schemes (GKS) [5–7], and the smoothed particle hydrodynamics (SPH) [8]. Among these methods, the LBE and GKS methods are specifically designed for CFD. The kinetic nature of the LBE and GKS has led to many distinctive advantages that distinguish them from the classical CFD methods, and a variety of successful applications have been achieved [9–18]. Particularly, the lattice BGK (LBGK) [19, 20] model and multiple-relaxation-time (MRT) model, as two popular standard LBE methods, have been successfully applied and well-accepted for incompressible NS solutions [21, 22]. With the improved collision model, the MRT has overcome the apparent defects in the LBGK [22,23]. Besides the standard LBE [24], which can be viewed as a special finite-difference scheme for the discrete velocity Boltzmann equation (DVBE) using a regular lattice associated with the discrete velocities, the LBE methods have many other variants. The extended LBE, which solve the DVBE using general finite-difference [25, 26], finite-volume (FV) [27–30], or finite-element methods [31], can release the close coupling of the mesh and discrete velocities. As a result, arbitrary meshes can be employed in these generalized LBE methods. However, the decoupling in the extended LBE also destroys the nice features of the standard LBE. For example, many of the existing FV-LBE methods suffer from large numerical dissipation and poor numerical stability [28, 29].

Recently, starting from the Boltzmann equation, a discrete unified gas-kinetic scheme (DUGKS) has been proposed for isothermal flow in all Knudsen regimes [7]. The DUGKS is a finite volume method, which combines the advantages of GKS in its flux modeling