

# Assessment of Heat Flux Prediction Capabilities of Residual Distribution Method: Application to Atmospheric Entry Problems

Jesús Garicano Mena<sup>1,2,\*</sup>, Raffaele Pepe<sup>1,3</sup>, Andrea Lani<sup>1</sup> and Herman Deconinck<sup>1,2</sup>

<sup>1</sup> von Karman Institute for Fluid Dynamics, Sint-Genesius-Rhode, Belgium.

<sup>2</sup> Université Libre de Bruxelles, Bruxelles, Belgium.

<sup>3</sup> Scuola di Ingegneria, Università degli Studi della Basilicata, Potenza, Italy.

Received 7 April 2014; Accepted (in revised version) 21 November 2014

---

**Abstract.** In the present contribution we evaluate the heat flux prediction capabilities of second-order accurate Residual Distribution (*RD*) methods in the context of atmospheric (re-)entry problems around blunt bodies. Our departing point is the computation of subsonic air flows (with air modeled either as an inert ideal gas or as chemically reacting and possibly out of thermal equilibrium gas mixture) around probe-like geometries, as those typically employed into high enthalpy wind tunnels. We confirm the agreement between the solutions obtained with the *RD* method and the solutions computed with other Finite Volume (*FV*) based codes.

However, a straightforward application of the same numerical technique to hypersonic cases involving strong shocks exhibits severe deficiencies even on a geometry as simple as a 2D cylinder. In an attempt to mitigate this problem, we derive new variants of *RD* schemes. A comparison of these alternative strategies against established ones allows us to derive a diagnose for the shortcomings observed in the traditional *RD* schemes.

**PACS:** 47.40.Ki, 47.70.Nd, 47.11.-j, 47.11.Fg

**Key words:** Unstructured mesh, hypersonic flows, thermo-chemical nonequilibrium, residual distribution schemes.

---

## 1 Introduction

The numerical simulation of hypersonic flows can still be challenging, especially on purely unstructured meshes, including only triangles in 2D or tetrahedra in 3D [1,2]. For

---

\*Corresponding author. *Email addresses:* [jesus.garicano.mena@vki.ac.be](mailto:jesus.garicano.mena@vki.ac.be) (J. Garicano Mena), [raffaele.pepe@unibas.it](mailto:raffaele.pepe@unibas.it) (R. Pepe), [lanivki.ac.be](mailto:lanivki.ac.be) (A. Lani), [deconinck@vki.ac.be](mailto:deconinck@vki.ac.be) (H. Deconinck)

those kind of problems, the complexity characterizing compressible flow aerodynamics has to be addressed together with additional high-temperature effects due to chemical (CNEQ) or thermal-chemical nonequilibrium (TCNEQ) phenomena (see [3]).

Among the emerging techniques for tackling those kind of simulations,  $RD^\dagger$  methods evince several advantageous features for the computation of compressible flows, including the following:

- a maximum principle (Local Extrema Diminishing) borrowed from  $FV$  schemes, which allows for capturing discontinuities monotonically;
- compact stencils on arbitrary unstructured meshes, due to the underlying finite element ( $FE$ ) formulation, allowing for obtaining second-order accurate solutions and guaranteeing an excellent performance for parallel calculations;
- less grid sensitivity which has been proven on simplicial meshes in comparison with standard  $FV$ , thanks to a built-in multidimensional dissipation property.

The reasons above motivate our interest in the applicability of  $RD$  schemes to the simulation of atmospheric entry flow problems. Despite a few promising examples of numerical prediction of surface heat fluxes with  $RD$  in hypersonic flows (e.g. in the case of a sharp nosed double cone configuration, as reported in [4–6]), the prediction of the convective heating induced by the hot gas generated in the shock layer downstream a strong bow shock is not yet fully mastered. This does not only apply to the method in question but also to standard  $FV$  schemes. Indeed, it turns out that standard  $RD$  methods are affected by the same numerical shock instabilities (e.g. the carbuncle [5,7,8]) observed in  $FV$  flow computations. As examples, the reader is referred to [9] or [2] for structured and unstructured grids, respectively.

This paper is organized as follows. After having recalled the governing equations and some basic concepts for  $RD$  methods, we investigate their capability to accurately predict wall heat flux distributions for smooth flow fields, i.e. in absence of shock discontinuities. Nonequilibrium effects are included in this first analysis only. As a second step, we investigate how these methods perform on a simple cylinder configuration with conditions taken from available literature, involving strong shocks in a non-reactive gas. In this case, since severe deficiencies are exhibited by existing  $RD$  schemes, new variants of those are introduced and their potential to correctly predict wall heat fluxes is analyzed.

## 2 Governing equations

The system of equations describing the flows of our interest accounts for the conservation of mass, momentum and energy. In compact, vector form it reads:

$$\frac{\partial \vec{U}}{\partial t} + \nabla \cdot \vec{F}^c = \nabla \cdot \vec{F}^d + \vec{S}, \quad (2.1)$$

---

<sup>†</sup> $RD$  methods are also known under the name of Fluctuation Splitting ( $FS$ ) schemes.